EC 224: Intermediate Microeconomics

Lecture Notes

Spring 2020

Contents

1 Course Introduction and Math Review

1.1 Course Overview

This course covers the core topics of intermediate microeconomics, building on principles learned in introductory economics. The main areas of study include:

- **Consumer Theory**: Utility and choice, demand curves, income and substitution effects, types of goods
- **Uncertainty**: Risk aversion, expected utility, insurance, and financial assets
- **Game Theory**: Strategic interaction, Nash equilibrium, mixed strategies, sequential games
- **Producer Theory**: Production functions, costs, and firm behavior
- **Market Structures**: Perfect competition, monopoly, and imperfect competition (Cournot, Bertrand, Stackelberg)

Key Point

The course is divided into three main exam sections:

1. **First Midterm**: Consumer theory (utility, demand, uncertainty) and game theory
2. **Second Midterm**: Production, costs, and perfect competition
3. **Final Exam**: Monopoly, imperfect competition, and comprehensive review

1.2 Mathematical Review: Functions of One Variable

A **function** describes how one variable depends on another. We write $Y = f(X)$ to indicate that Y depends on X, where X is the **independent variable** and Y is the **dependent variable**.

Example 1.1. Consider the relationship between labor hours (X) and wages earned (Y):

$$Y = f(X) = aX + b$$

This is a **linear function** where:

- a is the **slope** (wage rate per hour)
- b is the **intercept** (base pay when $X = 0$)

1.2.1 Linear Functions

For a linear function $Y = aX + b$:

- The slope a represents the change in Y for a one-unit change in X
- If $a > 0$, the function is upward sloping
- If $a < 0$, the function is downward sloping

- The derivative is constant: $\dfrac{dY}{dX} = a$

Example 1.2. Given $Y = 3 + 2X$:

- Intercept: When $X = 0$, $Y = 3$
- Slope: $\dfrac{dY}{dX} = 2$
- When $X = 1$, $Y = 5$
- X-intercept: When $Y = 0$, $X = -1.5$

1.2.2 Quadratic Functions

A **quadratic function** has the form:

$$Y = f(X) = aX^2 + bX + c$$

Property 1.1 (Properties of Quadratic Functions)**.**

- If $a > 0$: parabola opens upward (has a **minimum**)
- If $a < 0$: parabola opens downward (has a **maximum**)
- The derivative is: $\dfrac{dY}{dX} = 2aX + b$
- At the vertex (extremum): $\dfrac{dY}{dX} = 0 \Rightarrow X = -\dfrac{b}{2a}$

Example 1.3. Consider $Y = X^2 - 8X + 9$ (standard form with $a = 1$, $b = -8$, $c = 9$).

Converting to vertex form by completing the square:

$$\begin{aligned} Y &= X^2 - 8X + 9 \\ &= (X^2 - 8X + 16) - 16 + 9 \\ &= (X - 4)^2 - 7 \end{aligned}$$

This is now in **vertex form**: $Y = a(X - h)^2 + k$ where $(h, k) = (4, -7)$ is the vertex.

Key features:

- Vertex: $(4, -7)$ — this is the minimum since $a = 1 > 0$
- Y-intercept: When $X = 0$, $Y = 9$
- X-intercepts: When $Y = 0$:

$$\begin{aligned} (X - 4)^2 &= 7 \\ X - 4 &= \pm\sqrt{7} \\ X &= 4 \pm \sqrt{7} \approx 1.35 \text{ or } 6.65 \end{aligned}$$

1.3 Functions of Two Variables

Many economic relationships involve multiple variables. We write $Y = f(X, Z)$ to indicate that Y depends on both X and Z.

Example 1.4. Consider the function $Y = \sqrt{X \cdot Z}$ (a Cobb-Douglas form).
For different values of Y, we can find combinations of X and Z:

- If $Y = 1$: $X = 1, Z = 1$ or $X = 0.5, Z = 2$ or $X = 2, Z = 0.5$
- If $Y = 2$: $X = 2, Z = 2$ or $X = 1, Z = 4$
- If $Y = 3$: $X = 3, Z = 3$ or $X = 1, Z = 9$

1.4 Simultaneous Equations

Many economic problems require solving systems of equations to find equilibrium points.

Example 1.5. Solve the system:

$$\begin{cases} X + Y = 3 \\ X - Y = 1 \end{cases}$$

Solution: Adding the equations: $2X = 4 \Rightarrow X = 2$, then $Y = 1$.
Graphically, this is the intersection of:

- Line 1: $Y = -X + 3$ (slope $= -1$, intercept $= 3$)
- Line 2: $Y = X - 1$ (slope $= 1$, intercept $= -1$)

2 Utility and Choice

2.1 What is Utility?

Definition 2.1 (Utility)**. Utility** represents the satisfaction or well-being that an individual derives from consuming goods and services or engaging in economic activities.

Utility is a complex concept that depends on many factors:

- **Tastes and preferences**: Individual likes and dislikes
- **Personality**: Risk tolerance, patience, etc.
- **Affect and behavior**: Emotional state, habits
- **Love, security**: Non-material sources of satisfaction

Note

Ceteris Paribus ("other things being equal"): When analyzing utility, we hold all other factors constant to isolate the effect of specific goods on satisfaction.

2.2 The Utility Function

We represent utility mathematically as:

$$U = U(X, Y; \text{other things})$$

where X and Y are quantities of two goods, and "other things" represents all factors held constant.

For simplicity, we write:

$$U = U(X, Y)$$

This notation means utility depends on X and Y, with all other factors that affect utility held constant.

Key Point

Measuring Utility: Economists Daniel Kahneman and Angus Deaton have developed surveys to measure subjective well-being:

- "If your life were a ladder from 0 to 10, where do you see yourself now? In 5 years?"
- "Did you feel happy/sad/anxious/angry yesterday?"

Research using the ACA Medicaid expansion has examined how health insurance affects subjective well-being (SWB).

2.3 Assumptions About Preferences

For utility theory to work, we assume individuals have **rational preferences**—relatively well-defined preferences that satisfy three key axioms:

Definition 2.2 (Completeness)**.** For any two bundles A and B, exactly one of the following holds:

- $A \succ B$ (A is strictly preferred to B)
- $A \prec B$ (B is strictly preferred to A)
- $A \sim B$ (A and B are equally preferred—indifferent)

This means you are **capable of making decisions**—you can always compare any two options.

Definition 2.3 (Transitivity)**.** If $A \succ B$ and $B \succ C$, then $A \succ C$.
This ensures **consistency** in preferences—no preference cycles.

Definition 2.4 (More is Better (Non-satiation))**.** If bundle (X^*, Y^*) contains at least as much of both goods as bundle (X, Y), with strictly more of at least one good, then:

$$U(X^*, Y^*) > U(X, Y)$$

2.4 Indifference Curves

Definition 2.5 (Indifference Curve)**.** An **indifference curve** represents all combinations of two goods that provide the same level of utility to an individual.

Property 2.1 (Properties of Indifference Curves)**.**
1. **Downward sloping**: To maintain the same utility while getting more of one good, you must give up some of the other (due to "more is better")
2. **Higher curves = higher utility**: $U_3 > U_2 > U_1$
3. **Cannot cross**: If they crossed, transitivity would be violated
4. **Convex to the origin**: Reflects diminishing marginal rate of substitution

Example 2.1 (Hamburgers and Soft Drinks)**.** Consider points on an indifference curve:

Point	Hamburgers	Soft Drinks
A	6	2
B	4	3
C	3	4
D	2	6

All these bundles provide the same utility: $A \sim B \sim C \sim D$.

By transitivity and "more is better": Point E (at 4 hamburgers, 5 soft drinks) would be on a **higher** indifference curve since it has more of at least one good than points on curve U_1.

2.5 Marginal Rate of Substitution (MRS)

Definition 2.6 (Marginal Rate of Substitution)**.** The **MRS** measures the rate at which a consumer is willing to trade one good for another while maintaining the same level of utility:

$$MRS_{XY} = -\frac{\Delta Y}{\Delta X}\bigg|_{U=\text{constant}} = \frac{MU_X}{MU_Y}$$

The MRS equals the **absolute value of the slope** of the indifference curve.

Example 2.2 (Calculating MRS)**.** Moving along the indifference curve:

- $A \to B$: Give up 2 hamburgers to get 1 soft drink $\Rightarrow MRS = 2$
- $B \to C$: Give up 1 hamburger to get 1 soft drink $\Rightarrow MRS = 1$
- $C \to D$: Give up 0.5 hamburgers to get 1 soft drink $\Rightarrow MRS = 0.5$

Key Point

Diminishing MRS: As you consume more of good X (soft drinks), the amount of good Y (hamburgers) you're willing to give up for one more unit of X decreases. This reflects the idea that **balanced consumption bundles are generally preferred** to extreme ones.

2.6 Balance in Consumption

Consumers generally prefer **balanced bundles** over extreme ones because of diminishing MRS. This is why indifference curves are typically convex to the origin.

Note

Product Positioning: Companies use the concept of indifference curves when positioning products. Before tablets like the iPad, consumers chose between laptops (high portability, moderate power) and desktop computers (low portability, high power). The iPad created a new product category that offered a different combination of attributes.

2.7 Special Types of Preferences

Not all goods follow the standard convex indifference curve pattern:

2.7.1 Useless Goods

A good that provides no utility. Indifference curves are **vertical lines**—only the useful good matters.

2.7.2 Economic Bads

A good that reduces utility (e.g., pollution, houseflies). Indifference curves slope **upward**—you need more of the good good to compensate for more of the bad.

2.7.3 Perfect Substitutes

Goods that can be exchanged at a **constant rate**. Indifference curves are **straight lines** with constant MRS.

Example 2.3. Gallons of Exxon gas vs. gallons of Mobil gas—most consumers view these as interchangeable.

2.7.4 Perfect Complements

Goods consumed in **fixed proportions**. Indifference curves are **L-shaped**.

Example 2.4. Left shoes and right shoes—having 3 left shoes and 2 right shoes gives the same utility as having 2 left and 2 right shoes.

2.8 Practice Problem Solutions

Example 2.5 (Cobb-Douglas Utility)**.** Given $U = \sqrt{A \cdot B} = A^{1/2}B^{1/2}$, with initial bundle $A = 5$, $B = 80$:

(a) Calculate utility: $U = \sqrt{5 \times 80} = \sqrt{400} = 20$
(b) Find B when $A = 10$ and $U = 20$:

$$20 = \sqrt{10 \times B} \Rightarrow 400 = 10B \Rightarrow B = 40$$

(c) Find the MRS:

$$MRS = \frac{MU_A}{MU_B} = \frac{\partial U/\partial A}{\partial U/\partial B} = \frac{\frac{1}{2}A^{-1/2}B^{1/2}}{\frac{1}{2}A^{1/2}B^{-1/2}} = \frac{B}{A}$$

At point $A = 5$, $B = 80$: $MRS = \frac{80}{5} = 16$
At point $A = 10$, $B = 40$: $MRS = \frac{40}{10} = 4$

Example 2.6 (Linear (Perfect Substitutes) Utility)**.** Given $U = 3C + 4T$ (coffee and tea), with $P_C = 2$, $P_T = 3$, Income = \$12:

(a) The utility function is linear, so indifference curves are straight lines.
(b) MRS is constant: $MRS = \frac{MU_C}{MU_T} = \frac{3}{4}$
(c) With the budget constraint $2C + 3T = 12$:

- All tea ($C = 0$): $T = 4$, $U = 3(0) + 4(4) = 16$
- All coffee ($T = 0$): $C = 6$, $U = 3(6) + 4(0) = 18$

Optimal choice: Spend all income on coffee (corner solution) because $\frac{MU_C}{P_C} = \frac{3}{2} > \frac{MU_T}{P_T} = \frac{4}{3}$.

Example 2.7 (Perfect Complements)**.** Given $U = \min(2M, C)$ (milk and cookies):
The consumer always wants $2M = C$ (consumes in ratio $M : C = 1 : 2$).
Points on indifference curve $U = 2$:

- $M = 1$, $C = 2$ gives $U = \min(2, 2) = 2$ ✓
- $M = 1$, $C = 3$ gives $U = \min(2, 3) = 2$ ✓(extra C is wasted)
- $M = 2$, $C = 2$ gives $U = \min(4, 2) = 2$ ✓(extra M is wasted)

The kink of the L-shaped indifference curve occurs where $2M = C$.

3 Utility Maximization

3.1 The Consumer's Problem

The consumer's goal is to **choose the best affordable option**—maximize utility subject to a budget constraint.

Key Point

Constrained Optimization: Without scarce resources, there would be no limit on consumption. The budget constraint forces consumers to make trade-offs and allocate their limited income across goods.

3.2 The Budget Constraint

Definition 3.1 (Budget Constraint)**.** The **budget constraint** represents all combinations of goods that exactly exhaust the consumer's income:

$$P_X \cdot X + P_Y \cdot Y = I$$

where P_X and P_Y are prices, X and Y are quantities, and I is income.

3.2.1 Properties of the Budget Line

Solving for Y:

$$Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} \cdot X$$

- **Y-intercept**: $Y_{max} = \dfrac{I}{P_Y}$ (spend all income on good Y)
- **X-intercept**: $X_{max} = \dfrac{I}{P_X}$ (spend all income on good X)
- **Slope**: $-\dfrac{P_X}{P_Y}$ (the price ratio, or opportunity cost of X in terms of Y)

Note

- Points **on** the budget line: Consumer spends exactly all income (affordable, no waste)
- Points **below** the line: Affordable, but income is not fully spent
- Points **above** the line: Not affordable

3.3 Finding the Optimal Bundle

The optimal consumption bundle occurs where the highest attainable indifference curve is **tangent** to the budget line.

Theorem 3.1 (Optimality Condition)**.** At the optimal interior solution:

$$MRS = \frac{P_X}{P_Y}$$

or equivalently:

$$\frac{MU_X}{P_X} = \frac{MU_Y}{P_Y}$$

The marginal utility per dollar spent must be equal across all goods.

Key Point

Intuition: If $\frac{MU_X}{P_X} > \frac{MU_Y}{P_Y}$, the consumer gets more utility per dollar from good X, so they should buy more X and less Y. At the optimum, no reallocation can improve utility.

4 Individual Demand Curves

4.1 Optimal Choices Across Consumers

Different consumers with the same income and facing the same prices may choose **very different consumption bundles** depending on their preferences.

Example 4.1 (Three Consumers with Different Preferences)**.** Consider three consumers, each with income \$30, facing prices $P_H = \$3$ (hamburgers) and $P_D = \$1.50$ (soft drinks). The price ratio is $P_D/P_H = 0.5$.

- **Hungry Joe**: Prefers hamburgers heavily ⇒ buys mostly hamburgers (8 H, 4 D)
- **Thirsty Teresa**: More balanced preferences ⇒ buys mix (2 H, 16 D)
- **Extra-Thirsty Ed**: Strongly prefers drinks ⇒ buys mostly drinks (0 H, 20 D)

All three face the **same budget constraint** but reach different optimal points because their indifference curves have different shapes.

4.2 Corner Solutions for Special Preferences

4.2.1 Useless Goods

If a good provides no utility, the consumer spends nothing on it. The optimal choice is at a **corner** of the budget constraint.

4.2.2 Economic Bads

For economic bads (goods that reduce utility), the consumer minimizes consumption of the bad—again, a corner solution at zero consumption of the bad.

4.2.3 Perfect Substitutes

With perfect substitutes, the consumer **buys only the cheaper good** (corner solution), unless prices are exactly proportional to marginal utilities.

4.2.4 Perfect Complements

With perfect complements (L-shaped indifference curves), the consumer buys goods in **fixed proportions**. The optimal point is at the kink of the indifference curve.

4.3 Non-Linear Pricing

Sometimes prices change based on quantity purchased (quantity discounts, tiered pricing).

Example 4.2 (Quantity Discount)**.** Suppose bird seed costs \$4 per bag for the first 5 bags, but only \$3 per bag after that. With income = \$50:

- First 5 bags cost: $5 \times \$4 = \$20$
- Remaining \$30 buys: $\$30/\$3 = 10$ more bags
- Maximum purchase: 15 bags total

This creates a **kinked budget constraint**—the slope changes at the quantity threshold.

4.4 Composite Goods

A **composite good** represents aggregate spending on many items, allowing us to analyze choices between one specific good and "everything else."

Example 4.3. Instead of modeling choices among hundreds of goods, we can simplify to:

- X = Housing
- Y = "Everything else" (a composite good measured in dollars)

The price of the composite good is \$1 per unit (since it's measured in dollars).

4.5 The Demand Function

Definition 4.1 (Individual Demand Function)**.** The **demand function** shows the quantity demanded as a function of prices, income, and preferences:

$$Q_X^d = d_X(P_X, P_Y, I; \text{preferences})$$

Similarly for good Y:

$$Q_Y^d = d_Y(P_X, P_Y, I; \text{preferences})$$

Preferences are held constant when analyzing demand.

4.6 Homogeneity of Degree Zero

Theorem 4.1 (Homogeneity of Demand)**.** Individual demand is **homogeneous of degree zero** in prices and income. This means:

$$d_X(tP_X, tP_Y, tI) = d_X(P_X, P_Y, I) \quad \text{for any } t > 0$$

Key Point

No Money Illusion: If all prices and income double, the budget constraint is unchanged:

$$P_X X + P_Y Y = I \quad \Leftrightarrow \quad 2P_X X + 2P_Y Y = 2I$$

Therefore, demand depends only on **relative prices** (P_X/P_Y) and **real income** (I/P), not nominal values.

4.7 Effects of Income Changes

When income increases (prices constant), the budget constraint shifts **outward** parallel to itself.

Definition 4.2 (Normal Good)**.** A good is **normal** if demand increases when income increases:

$$\frac{\partial Q_X}{\partial I} > 0$$

Definition 4.3 (Inferior Good)**.** A good is **inferior** if demand decreases when income increases:

$$\frac{\partial Q_X}{\partial I} < 0$$

Example 4.4 (Engel's Law)**. Engel's Law**: As income rises, the **fraction** of income spent on food decreases.

Data from U.S. consumers (2009):

Item	<\$70K	\$70K–\$100K	>\$100K
Food	14.1%	13.1%	11.4%
Housing	37.0%	33.2%	31.6%
Health + Pensions	13.9%	18.2%	20.1%

Food is a **necessity** (income elasticity < 1), while health/pensions are **luxuries** (income elasticity > 1).

5 Income and Substitution Effects

5.1 Decomposing Price Changes

When the price of a good changes, two effects occur simultaneously:

Definition 5.1 (Substitution Effect)**.** The **substitution effect** captures the change in quantity demanded due to the change in **relative prices**, holding utility constant.
If $P_X \downarrow$: Good X becomes relatively cheaper $\Rightarrow$ consume more X, less Y

Definition 5.2 (Income Effect)**.** The **income effect** captures the change in quantity demanded due to the change in **purchasing power** (real income).
If $P_X \downarrow$: Real income increases $\Rightarrow$ can afford more of both goods (if normal)

5.2 Graphical Decomposition

To separate the two effects:

1. Start at initial optimum on U_1
2. **Substitution effect**: Find the point on U_1 that would be optimal at the new price ratio (hypothetical budget line tangent to U_1 with new slope)
3. **Income effect**: Move from this hypothetical point to the actual new optimum on U_2

5.3 Normal Goods: Both Effects Reinforce

For **normal goods**, both effects work in the same direction:
If $P_X \downarrow$:

- Substitution effect: $X \uparrow$ (X is relatively cheaper)
- Income effect: $X \uparrow$ (higher real income $\Rightarrow$ more of normal good)
- Total effect on X: **Unambiguously positive**

Note

For good Y, the effects work in opposite directions:

- Substitution effect: $Y \downarrow$ (substitute toward cheaper X)
- Income effect: $Y \uparrow$ (if Y is normal)
- Total effect on Y: **Ambiguous**

5.4 Special Cases

5.4.1 Perfect Complements

With perfect complements, there is **no substitution effect**—goods must be consumed in fixed proportions regardless of relative prices. Only the income effect matters.

5.4.2 Perfect Substitutes

With perfect substitutes, there is **only a substitution effect** (for non-marginal price changes). The consumer switches entirely to the cheaper good.

5.5 Inferior Goods

For **inferior goods**, the income and substitution effects work in **opposite directions**:
If $P_X \downarrow$ and X is inferior:

- Substitution effect: $X \uparrow$ (X is relatively cheaper)
- Income effect: $X \downarrow$ (higher real income $\Rightarrow$ less of inferior good)
- Total effect: Depends on which effect dominates

Key Point

For most inferior goods, the **substitution effect dominates**, so the Law of Demand still holds ($P \downarrow \Rightarrow Q \uparrow$).
The exception is the rare **Giffen good**.

5.6 Giffen's Paradox

Definition 5.3 (Giffen Good)**.** A **Giffen good** is an inferior good where the income effect is so large that it **dominates** the substitution effect, causing demand to move in the same direction as price:

$$P_X \downarrow \Rightarrow Q_X \downarrow \quad \text{(violates Law of Demand)}$$

Note

Giffen goods are extremely rare and require:

1. The good must be inferior
2. The good must be a substantial part of the budget
3. There must be few substitutes

Historical example: Potatoes during the Irish Potato Famine (1840s). When potato prices fell, poor families could afford more meat, so they bought *less* potatoes.

5.7 The Lump-Sum Principle

Theorem 5.1 (Lump-Sum Principle)**.** Taxes imposed on **income** (lump-sum taxes) have smaller welfare costs than taxes imposed on **specific goods** (excise taxes) that raise the same revenue.

Key Point

Why? An excise tax distorts relative prices, causing both:

- Income effect (less purchasing power)
- Substitution effect (distorted choices away from taxed good)

A lump-sum tax only causes an income effect, so it's less distortionary.

Example 5.1 (Comparing Tax Types)**.** Suppose income = \$150, and initially $P_S = P_C = \$1$ (soda and coffee).

Excise tax: \$0.60 tax per soda $\Rightarrow P_S' = \$1.60$

- Consumer buys 30 sodas, pays \$18 in tax
- Utility: $U_1 = \sqrt{75 \times 75} \approx 75$

Lump-sum tax: Take \$18 directly from income

- New income = \$132, prices unchanged
- Consumer optimizes with full budget at undistorted prices
- Utility: $U_2 = \sqrt{66 \times 66} = 66$ (but no substitution distortion)

With the lump-sum tax, $U_3 > U_1$ because no substitution distortion occurs.

6 Consumer Surplus and Market Demand

6.1 Consumer Surplus

Definition 6.1 (Consumer Surplus)**. Consumer surplus** (CS) is the difference between what consumers are **willing to pay** for a good and what they **actually pay**:

$$CS = \text{Willingness to Pay} - \text{Price Paid}$$

It measures the monetary value of the utility gain from being able to purchase at the market price.

6.2 Willingness to Pay and the Demand Curve

The **demand curve** represents the consumer's **marginal willingness to pay** for each additional unit.

Example 6.1 (T-Shirts)**.** A consumer's willingness to pay for t-shirts:

Unit	WTP
1st t-shirt	\$15
15th t-shirt	\$9
20th t-shirt	\$7

If the market price is \$7:

- Consumer buys 20 t-shirts
- CS on 20th shirt = \$7 - \$7 = \$0
- CS on 1st shirt = \$15 - \$7 = \$8
- Total CS = Area of triangle above price, below demand curve

6.3 Calculating Consumer Surplus

For a linear demand curve, consumer surplus is the **triangular area** between the demand curve and the price line:

$$CS = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times Q \times (P_{max} - P)$$

Example 6.2. If demand is linear with maximum WTP = \$15, market price = \$7, and quantity = 20:

$$CS = \frac{1}{2} \times 20 \times (\$15 - \$7) = \frac{1}{2} \times 20 \times \$8 = \$80$$

6.4 Market Demand

Definition 6.2 (Market Demand)**. Market demand** is the **horizontal sum** of all individual demand curves:

$$Q^{market} = \sum_{i=1}^{n} Q_i(P)$$

At each price, we add up the quantities demanded by all consumers.

Note

All consumers face the **same market price**, but may demand different quantities based on their individual preferences and incomes.

7 Elasticity of Demand

7.1 Price Elasticity of Demand

Definition 7.1 (Price Elasticity of Demand)**.** The **price elasticity of demand** measures the percentage change in quantity demanded in response to a 1% change in price:

$$\varepsilon_{Q,P} = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q/Q}{\Delta P/P}$$

Note

Price elasticity is typically **negative** (Law of Demand), but we often discuss its absolute value $|\varepsilon_{Q,P}|$.

7.2 Interpreting Elasticity Values

- $|\varepsilon| > 1$: **Elastic** demand—quantity responds more than proportionally to price
- $|\varepsilon| = 1$: **Unit elastic**—quantity responds proportionally
- $|\varepsilon| < 1$: **Inelastic** demand—quantity responds less than proportionally

Example 7.1.
- $\varepsilon = -2$: A 10% price increase causes a 20% decrease in quantity (elastic)
- $\varepsilon = -0.5$: A 10% price increase causes a 5% decrease in quantity (inelastic)

7.3 Determinants of Elasticity

Demand tends to be **more elastic** when:

1. More **close substitutes** are available
2. The good is a **luxury** rather than a necessity
3. More **time** is available to adjust
4. The good represents a **larger share** of the budget

7.4 Elasticity and Total Revenue

Total spending (or total revenue for sellers) is:

$$TS = P \times Q$$

The relationship between price changes and total spending depends on elasticity:

	$\|\varepsilon\| > 1$ (Elastic)	$\|\varepsilon\| = 1$	$\|\varepsilon\| < 1$ (Inelastic)
$P\uparrow$	$TS\downarrow$	TS unchanged	$TS\uparrow$
$P\downarrow$	$TS\uparrow$	TS unchanged	$TS\downarrow$

Key Point

Intuition:

- If demand is elastic, quantity changes dominate $\Rightarrow$ lower price increases revenue
- If demand is inelastic, price changes dominate $\Rightarrow$ higher price increases revenue

7.5 Income Elasticity of Demand

Definition 7.2 (Income Elasticity)**.** The **income elasticity of demand** measures the percentage change in quantity demanded in response to a 1% change in income:

$$\varepsilon_{Q,I} = \frac{\%\Delta Q}{\%\Delta I}$$

- $\varepsilon_{Q,I} > 0$: **Normal good**
- $\varepsilon_{Q,I} < 0$: **Inferior good**
- $\varepsilon_{Q,I} > 1$: **Luxury good** (income elastic)
- $0 < \varepsilon_{Q,I} < 1$: **Necessity** (income inelastic)

7.6 Cross-Price Elasticity of Demand

Definition 7.3 (Cross-Price Elasticity)**.** The **cross-price elasticity** measures how the quantity demanded of good X responds to changes in the price of good Y:

$$\varepsilon_{Q_X,P_Y} = \frac{\%\Delta Q_X}{\%\Delta P_Y}$$

- $\varepsilon_{Q_X,P_Y} > 0$: X and Y are **substitutes** (higher $P_Y \Rightarrow$ more X)
- $\varepsilon_{Q_X,P_Y} < 0$: X and Y are **complements** (higher $P_Y \Rightarrow$ less X)

7.7 Practice Problem: Elasticity Calculation

Example 7.2. Given demand: $Q = 20 - P$
At $P_1 = 10$: $Q_1 = 10$, $TS_1 = 100$
At $P_2 = 15$: $Q_2 = 5$, $TS_2 = 75$
Calculate elasticity using midpoint method:

$$\%\Delta Q = \frac{Q_2 - Q_1}{(Q_1 + Q_2)/2} = \frac{5 - 10}{7.5} = -\frac{5}{7.5} \approx -67\%$$

$$\%\Delta P = \frac{P_2 - P_1}{(P_1 + P_2)/2} = \frac{15 - 10}{12.5} = \frac{5}{12.5} = 40\%$$

$$\varepsilon = \frac{-67\%}{40\%} \approx -1.67$$

Since $|\varepsilon| > 1$, demand is **elastic** at this price range.
Note: $P \uparrow$ and $TS \downarrow$ (from 100 to 75), confirming elastic demand.

Example 7.3 (Revenue-Maximizing Price)**.** For $Q = 20 - P$:

$$TS = P \times Q = P(20 - P) = 20P - P^2$$

To maximize:

$$\frac{d(TS)}{dP} = 20 - 2P = 0 \Rightarrow P^* = 10$$

At $P = 10$: $Q = 10$, and $TS = 100$ is maximized.

At the revenue-maximizing point, $|\varepsilon| = 1$ (unit elastic).

8 Uncertainty and Risk

8.1 Introduction to Uncertainty

Uncertainty affects many economic decisions:

- Investment choices
- Insurance purchases
- Career decisions
- Health behaviors

Key questions we address:

1. Why do people **dislike risk**?
2. How can we **reduce risk**?

8.2 Random Variables and Expected Value

Definition 8.1 (Random Variable)**.** A **random variable** Y summarizes the outcome of an event that involves uncertainty. It can take different values with different probabilities.

Example 8.1.

- $Y \in \{0, 1\}$: Success/Failure on an exam
- $X \in \{0, 100\}$: Exam performance (score)

Definition 8.2 (Expected Value)**.** The **expected value** is the long-run average outcome:

$$E[Y] = \mu_Y = \sum_i Y_i \cdot P_i$$

where Y_i are the possible outcomes and P_i are their probabilities.

8.3 Fair Gambles and Risk Aversion

Example 8.2 (A Simple Gamble)**.** Consider a gamble with two outcomes:

- Win \$10 with probability 50%
- Lose \$1 with probability 50%

Expected value:

$$E[X] = 10 \times 0.5 + (-1) \times 0.5 = 5 - 0.5 = \$4.50$$

Question: How much would you pay to play this gamble?

Definition 8.3 (Fair Gamble)**.** A **fair gamble** has an expected value of zero—on average, you neither gain nor lose.

Definition 8.4 (Risk Aversion)**.** A person is **risk averse** if they refuse to accept a fair gamble. Most people are risk averse—they prefer a certain outcome to a gamble with the same expected value.

8.4 Utility and Risk Aversion

Risk aversion arises from **diminishing marginal utility of income**. As income increases, utility increases but at a decreasing rate.

Example 8.3 (Concave Utility Function)**.** Let $U = \sqrt{\text{Income}}$

First derivative (marginal utility):

$$\frac{dU}{dI} = \frac{1}{2}I^{-1/2} > 0 \quad \text{(utility increases with income)}$$

Second derivative:

$$\frac{d^2U}{dI^2} = -\frac{1}{4}I^{-3/2} < 0 \quad \text{(diminishing marginal utility)}$$

The negative second derivative means the utility function is **concave**—this generates risk aversion.

8.5 Expected Utility vs. Utility of Expected Value

Example 8.4 (Comparing Risky and Certain Outcomes)**.** Suppose $U = \sqrt{I}$ and consider two scenarios with the same expected income of \$35,000:

Scenario 1: Certain income

- Income = \$35,000 with certainty
- $U = \sqrt{35,000} = 187.08$
- $E[U] = 187.08$

Scenario 2: Risky income (moderate risk)

- Income = \$30,000 with 50% probability
- Income = \$40,000 with 50% probability
- $E[I] = 0.5 \times 30,000 + 0.5 \times 40,000 = \$35,000$
- $E[U] = 0.5 \times \sqrt{30,000} + 0.5 \times \sqrt{40,000}$
- $E[U] = 0.5 \times 173.21 + 0.5 \times 200 = 186.60$

Scenario 3: Risky income (high risk)

- Income = \$20,000 with 50% probability
- Income = \$50,000 with 50% probability
- $E[I] = 0.5 \times 20,000 + 0.5 \times 50,000 = \$35,000$
- $E[U] = 0.5 \times \sqrt{20,000} + 0.5 \times \sqrt{50,000}$
- $E[U] = 0.5 \times 141.42 + 0.5 \times 223.61 = 182.51$

Key Point

Key Result: $E[U(\text{certain})] > E[U(\text{risky})]$ even when $E[I]$ is the same!

$$U(E[I]) > E[U(I)] \quad \text{for risk-averse individuals}$$

This is **Jensen's Inequality** for concave functions.

8.6 Certainty Equivalent and Risk Premium

Definition 8.5 (Certainty Equivalent)**.** The **certainty equivalent** is the guaranteed amount that provides the same expected utility as a risky prospect:

$$U(CE) = E[U(I)]$$

Definition 8.6 (Risk Premium)**.** The **risk premium** is the maximum amount a person would pay to eliminate risk:

$$\text{Risk Premium} = E[I] - CE$$

Example 8.5 (Calculating Risk Premium)**.** For Scenario 3 above with $E[U] = 182.51$:
Find CE such that $U(CE) = 182.51$:

$$\sqrt{CE} = 182.51 \implies CE = 182.51^2 = \$33,310$$

Risk premium:

$$\text{Risk Premium} = \$35,000 - \$33,310 = \$1,690$$

This person would pay up to $1,690 to avoid the risk.

8.7 Properties of Risk Aversion

1. **Steeper curve $\Rightarrow$ more risk averse**: Greater curvature in the utility function means stronger risk aversion
2. **Linear utility $\Rightarrow$ risk neutral**: If $U = aI + b$, the person is indifferent between fair gambles and certain outcomes
3. **Smaller gambles $\Rightarrow$ lower risk premium**: For small risks, people behave approximately risk-neutral

8.8 Reducing Risk: Insurance

How to reduce risk? Create a market that puts a price on risk.

Definition 8.7 (Fair Insurance Premium)**.** A **fair insurance premium** equals the expected loss:

$$\text{Fair Premium} = E[\text{Loss}]$$

Example 8.6 (Insurance Decision)**.** Consider a person with:

- Current wealth: $35,000
- 50% chance of $15,000 loss (illness, accident, etc.)

- Utility function: $U = \sqrt{I}$

Without insurance:

- Income: \$20,000 or \$35,000 (each with 50%)
- $E[U] = 0.5\sqrt{20,000} + 0.5\sqrt{35,000} = 164.25$

Fair premium $= 0.5 \times \$15,000 = \$7,500$
With fair insurance:

- Pay \$7,500 premium, get \$15,000 if loss occurs
- Guaranteed income: \$35,000 - \$7,500 = \$27,500
- $U = \sqrt{27,500} = 165.83$

Since $165.83 > 164.25$, the person **buys insurance**.
Maximum premium the person would pay:

$$\sqrt{CE} = 164.25 \implies CE = \$26,978$$

$$\text{Max Premium} = \$35,000 - \$26,978 = \$8,022$$

Note

Risk-averse individuals will buy insurance as long as premiums are reasonable (not too far above the fair premium).

8.9 Problems in Insurance Markets

Insurance markets can fail due to **information asymmetry** between buyers and sellers:

8.9.1 Adverse Selection

Definition 8.8 (Adverse Selection)**. Adverse selection** occurs when those most likely to claim (highest risk) are most likely to buy insurance, driving up premiums and potentially causing market failure.

Example 8.7. Sick people are more likely to buy health insurance ⇒ insurance pool becomes riskier ⇒ premiums rise ⇒ healthy people drop out ⇒ "death spiral"

8.9.2 Moral Hazard

Definition 8.9 (Moral Hazard)**. Moral hazard** occurs when people act differently (take more risks) after transferring risk to an insurer.

Example 8.8. After buying car insurance, a driver may be less careful because they don't bear the full cost of accidents.

8.10 Reducing Risk: Diversification

Definition 8.10 (Diversification)**. Diversification** reduces risk by spreading investments across multiple assets whose returns are not perfectly correlated.

Example 8.9 (Diversification Benefit)**.** Compare two investment strategies:

Strategy i: Single asset

- Income: \$20,000 or \$50,000 (each with 50%)
- $E[I] = \$35,000$
- $E[U] = 0.5\sqrt{20,000} + 0.5\sqrt{50,000} = 182.5$

Strategy ii: Diversified portfolio

- Income: \$20,000 (25%), \$35,000 (50%), \$50,000 (25%)
- $E[I] = 0.25 \times 20,000 + 0.5 \times 35,000 + 0.25 \times 50,000 = \$35,000$
- $E[U] = 0.25\sqrt{20,000} + 0.5\sqrt{35,000} + 0.25\sqrt{50,000} = 184.8$

Same expected income, but diversification yields **higher expected utility**!

Key Point

Diversification works best when:

- Assets are not perfectly correlated
- You can spread across many different assets
- Transaction costs are low

8.11 Options and Flexibility

Definition 8.11 (Option Contract)**.** An **option contract** offers the right, but not the obligation, to buy or sell an asset at a specified price within a specified time period.

Key features that determine option value:

- **Transaction**: What asset, what quantity
- **Duration**: How long the option is valid
- **Strike Price**: The price at which you can buy/sell

8.11.1 Types of Options

- **Call Option**: Right to **buy** at strike price—profits when price rises
- **Put Option**: Right to **sell** at strike price—profits when price falls

Note

Options provide a way to reduce risk while maintaining upside potential. The buyer pays a premium for this flexibility.

8.12 Asset Pricing: Risk vs. Return

Key Point

Risk-Return Tradeoff: Assets with higher risk should offer higher expected returns to compensate investors for bearing that risk.

A risk-averse investor chooses portfolios along the **efficient frontier**—combinations that maximize expected return for a given level of risk.

9 Game Theory

9.1 Introduction to Strategic Interaction

In many economic situations, your best action depends on what others do. **Game theory** provides tools to analyze these strategic interactions.

Note

Game theory emerged in the 1950s during the Cold War era. Key contributors include John von Neumann and John Nash (who won the Nobel Prize).

9.2 Elements of a Game

1. **Players**: Decision makers (2, 3, ... n players)
2. **Strategies**: Each player's available choices
   - In simple games, strategies = actions
   - May be contingent on other players' actions
   - Can involve randomization
3. **Payoffs**: What players care about (utility, income, profit, etc.)
4. **Information**: What do players know?
   - Common knowledge
   - Sequential or simultaneous moves
   - Complete or incomplete information

9.3 Equilibrium Concepts

Definition 9.1 (Best Response)**.** A **best response** is a strategy that produces the highest payoff among all possible strategies for a player, *conditional on the other player's strategy.*

Definition 9.2 (Nash Equilibrium)**.** A **Nash equilibrium** is a set of strategies (one for each player) such that each player's strategy is a best response to the other players' strategies.

At a Nash equilibrium, **no player has an incentive to deviate** unilaterally.

Theorem 9.1 (Nash's Theorem)**.** Every finite game has at least one Nash equilibrium (possibly in mixed strategies).

9.4 The Prisoner's Dilemma

The most famous game in game theory illustrates how individual rationality can lead to collectively suboptimal outcomes.

Example 9.1 (Prisoner's Dilemma)**.** Two criminals are arrested and interrogated separately. Each can either **Confess** or remain **Silent**.

Payoff Matrix (years in prison, negative = bad):

Player B

Player A		Confess	Silent
	Confess	$-3, -3$	$-1, -10$
	Silent	$-10, -1$	$-2, -2$

Analysis:

- If B confesses: A gets -3 (confess) vs -10 (silent) $\Rightarrow$ A confesses
- If B is silent: A gets -1 (confess) vs -2 (silent) $\Rightarrow$ A confesses

Confess is a dominant strategy for both players—it's best regardless of what the other player does.

Definition 9.3 (Dominant Strategy)**.** A **dominant strategy** is one that is best for a player regardless of what other players choose.

Key Point

- Nash equilibrium: (Confess, Confess) with payoffs $(-3, -3)$
- But (Silent, Silent) would give $(-2, -2)$—better for both!
- **Individual rationality leads to collective irrationality**
- Cooperation would help both, but is not sustainable without enforcement

Note

Every dominant strategy equilibrium is a Nash equilibrium, but not vice versa.

9.5 Pure vs. Mixed Strategies

Definition 9.4 (Pure Strategy)**.** A **pure strategy** is a single action played with certainty.

Definition 9.5 (Mixed Strategy)**.** A **mixed strategy** involves randomly selecting from several possible actions according to specified probabilities.

9.6 Matching Pennies: A Game with No Pure Strategy Equilibrium

Example 9.2 (Matching Pennies)**.** Two players simultaneously choose Heads or Tails.

Player B

Player A		Heads	Tails
	Heads	$+1, -1$	$-1, +1$
	Tails	$-1, +1$	$+1, -1$

No pure strategy Nash equilibrium exists!

- A wants to *match* B's choice
- B wants to *mismatch* A's choice

- Any pure strategy can be exploited

Mixed strategy equilibrium: Both players randomize 50% Heads, 50% Tails.
Expected payoffs:

$$E[\text{Payoff}_A] = \frac{1}{4}(+1) + \frac{1}{4}(-1) + \frac{1}{4}(-1) + \frac{1}{4}(+1) = 0$$

$$E[\text{Payoff}_B] = 0$$

Neither player can improve by changing strategy unilaterally.

Key Point

Why randomize? Randomization prevents your opponent from exploiting predictable behavior. This is why penalty kicks in soccer involve mixed strategies!

9.7 Battle of the Sexes: Multiple Equilibria

Example 9.3 (Battle of the Sexes)**.** A couple wants to spend the evening together but has different preferences:

		Player B (Husband)	
		Ballet	Boxing
Player A (Wife)	Ballet	$\underline{2}, \underline{1}$	$0, 0$
	Boxing	$0, 0$	$\underline{1}, \underline{2}$

Two pure strategy Nash equilibria:

1. (Ballet, Ballet): Payoffs $(2, 1)$
2. (Boxing, Boxing): Payoffs $(1, 2)$

Both prefer being together to being apart, but they disagree on where to go.

Definition 9.6 (Best Response Function)**.** The **best response function** specifies the payoff-maximizing choice for each possible action the other player can take.

Note

In Battle of the Sexes:

- If Wife believes Husband goes to Ballet $\Rightarrow$ she goes to Ballet
- If Wife believes Husband goes to Boxing $\Rightarrow$ she goes to Boxing

The equilibrium depends on **expectations/beliefs** about the other player.

9.8 Mixed Strategy Equilibrium in Battle of the Sexes

Let's compute the mixed strategy Nash equilibrium for the Battle of the Sexes game.

Let h = probability that Husband goes to Ballet, and w = probability that Wife goes to Ballet.

Wife's Decision:

$$E[\text{Payoff}_W|\text{Ballet}] = h \times 2 + (1-h) \times 0 = 2h$$
$$E[\text{Payoff}_W|\text{Boxing}] = h \times 0 + (1-h) \times 1 = 1-h$$

Wife chooses Ballet if $2h > 1-h$, i.e., if $h > 1/3$.

Husband's Decision:

$$E[\text{Payoff}_H|\text{Ballet}] = w \times 1 + (1-w) \times 0 = w$$
$$E[\text{Payoff}_H|\text{Boxing}] = w \times 0 + (1-w) \times 2 = 2-2w$$

Husband chooses Ballet if $w > 2-2w$, i.e., if $w > 2/3$.

Key Point

Three Nash Equilibria:

1. **Pure**: $(w = 1, h = 1)$ — Both go to Ballet
2. **Pure**: $(w = 0, h = 0)$ — Both go to Boxing
3. **Mixed**: $(w = 2/3, h = 1/3)$ — Wife plays Ballet with prob 2/3, Husband with prob 1/3

Expected Payoffs at Mixed Equilibrium:

$$P(\text{Ballet, Ballet}) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$
$$P(\text{Ballet, Boxing}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$
$$P(\text{Boxing, Ballet}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$
$$P(\text{Boxing, Boxing}) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

Total expected payoff $= \frac{2}{9}(3) + \frac{4}{9}(0) + \frac{1}{9}(0) + \frac{2}{9}(3) = \frac{12}{9} = 1.33$

Note: The mixed equilibrium yields **lower** expected payoffs than either pure equilibrium!

9.9 Sequential Games

In **sequential games**, players move in order, and later players observe earlier moves before deciding.

Definition 9.7 (Extensive Form)**.** The **extensive form** (game tree) represents:

- The order of moves
- What each player knows when they move
- The payoffs at each terminal node

Example 9.4 (Sequential Battle of the Sexes)**.** Suppose Wife moves first, then Husband observes and responds.

Wife has 2 strategies: {Ballet, Boxing}
Husband has 4 **contingent strategies**:

1. Always Ballet: (Ballet|Ballet, Ballet|Boxing)
2. Match Wife: (Ballet|Ballet, Boxing|Boxing)
3. Oppose Wife: (Boxing|Ballet, Ballet|Boxing)
4. Always Boxing: (Boxing|Ballet, Boxing|Boxing)

Normal Form:

	Strategy 1	Strategy 2	Strategy 3	Strategy 4
Ballet	$\underline{2}, \underline{1}$	$\underline{2}, \underline{1}$	$0, 0$	$0, 0$
Boxing	$0, 0$	$\underline{1}, \underline{2}$	$0, 0$	$\underline{1}, \underline{2}$

Multiple Nash equilibria exist, but some involve **non-credible threats**.

Definition 9.8 (Non-Credible Threat)**.** A **non-credible threat** is a strategy that a player would not actually follow through on if called upon to act, because it's not in their self-interest at that point.

9.10 Subgame Perfect Equilibrium

Definition 9.9 (Proper Subgame)**.** A **proper subgame** starts at a single decision node and includes everything branching out from it.

Definition 9.10 (Subgame Perfect Equilibrium (SPE))**.** A **subgame perfect equilibrium** is a set of strategies that form a Nash equilibrium in **every proper subgame**.

SPE eliminates non-credible threats.

Key Point

Backward Induction: To find SPE:

1. Start at the final decision nodes
2. Determine optimal choices at each
3. Work backwards, replacing each subgame with its equilibrium payoff
4. Continue until reaching the initial node

Example 9.5 (Cuban Missile Crisis (Simplified))**.** Model the 1962 confrontation between JFK and Khrushchev:

Players: JFK (USA), Khrushchev (USSR)
JFK's options: Do Nothing, Air Strike, Naval Blockade
Khrushchev's options: Place Missiles or not; Acquiesce or Escalate
Utility Rankings:
For Khrushchev:

1. Missiles Allowed (best)
2. Status Quo
3. Acquiesce to Blockade
4. Acquiesce to Air Strike (loses resources)
5. Escalate after Air Strike (risk of nuclear war, but US is aggressor)
6. Escalate after Blockade (worst—risk of nuclear war, USSR is aggressor)

For JFK:

1. Blockade → Soviets Remove Missiles (best)
2. Air Strike → Soviets Acquiesce
3. Status Quo
4. Allow Missiles
5. Escalate after Blockade
6. Escalate after Air Strike (worst)

Using backward induction: The SPE predicts Blockade → Acquiesce, which is what historically occurred.

9.11 Repeated Games

When the same game is played multiple times, new equilibria can emerge.

Definition 9.11 (Trigger Strategy)**.** A **trigger strategy** involves cooperating until the other player defects, then punishing by defecting forever (or for some period).

Example 9.6 (Finitely Repeated Prisoner's Dilemma)**.** If the Prisoner's Dilemma is played exactly 3 times:

Backward Induction:

- Period 3: Last period, so both Confess (dominant strategy)
- Period 2: Period 3 outcome is fixed, so maximize Period 2 ⇒ Confess
- Period 1: Same logic ⇒ Confess

Result: Unique SPE is (Confess, Confess) in every period.

Key Point

Infinitely Repeated Games: If the game continues with probability g after each round, cooperation can be sustained.
With **Grim Trigger Strategy** (cooperate until defection, then always defect):
Cooperation payoff: $(-2)(1+g+g^2+...)=\frac{-2}{1-g}$
Defection payoff: $(-1)+(-3)(g+g^2+...)=-1+\frac{-3g}{1-g}$
Cooperation is sustainable if:

$$\frac{-2}{1-g}\geq -1+\frac{-3g}{1-g}$$

Solving: $g\geq\frac{1}{2}$
If there's at least a 50% chance of playing again, cooperation can be an equilibrium!

9.12 Tragedy of the Commons

The **Tragedy of the Commons** illustrates how individual optimization leads to over-exploitation of shared resources.

Example 9.7 (Grazing on Common Land)**.** Two farmers share a common pasture. Each chooses how many sheep to graze (S_A and S_B).

Total benefit for farmer i:

$$TB_i=S_i(120-S_A-S_B)$$

Marginal cost of grazing is zero.

Nash Equilibrium:

Each farmer maximizes their own benefit:

$$MB_A=120-2S_A-S_B=0\Rightarrow S_A=60-\frac{1}{2}S_B$$
$$MB_B=120-2S_B-S_A=0\Rightarrow S_B=60-\frac{1}{2}S_A$$

Solving: $S_A^*=S_B^*=40$
Total benefit: $TB_A=40(120-80)=1600$, $TB_B=1600$
Social Welfare $=3200$
Cooperative Solution (maximize joint benefit):

$$TB=S(120-S)\Rightarrow MB=120-2S=0\Rightarrow S^*=60$$

If they split equally: $S_A=S_B=30$
$TB_A=30(120-60)=1800$, $TB_B=1800$
Social Welfare $=3600$ (higher than Nash!)
But cooperation is not credible: If B commits to 30, A's best response:

$$S_A=60-\frac{1}{2}(30)=45$$

$TB_A=45(120-75)=2025>1800$

Each has incentive to cheat!

Key Point

The Tragedy of the Commons shares the same structure as the Prisoner's Dilemma:

- Individual rationality leads to collective irrationality
- The Nash equilibrium is inefficient
- Solutions require: property rights, regulation, or repeated interaction

10 Review Problems and Solutions

10.1 Utility Maximization Problems

Example 10.1 (Perfect Complements)**.** Given $U = \min(2H, S)$ with $MU_H = 10$, $MU_S = 2$:

For perfect complements, consume where $2H = S$.

The MRS is undefined at the kink. The consumer always chooses the bundle where $2H = S$, regardless of prices (as long as both goods are consumed).

Example 10.2 (Cobb-Douglas Optimization)**.** Given $U = C^{1/2}R^{1/2}$, $P_C = 1$, $P_R = 2$, $I = 20$:

Marginal utilities:

$$MU_C = \frac{1}{2}C^{-1/2}R^{1/2}$$
$$MU_R = \frac{1}{2}C^{1/2}R^{-1/2}$$

Optimality condition:

$$MRS = \frac{MU_C}{MU_R} = \frac{R}{C} = \frac{P_C}{P_R} = \frac{1}{2}$$

So $C = 2R$.

Budget constraint: $1 \cdot C + 2 \cdot R = 20$

Substituting: $2R + 2R = 20 \Rightarrow R = 5$, $C = 10$

Maximum utility: $U = \sqrt{10 \times 5} = \sqrt{50} \approx 7.07$

10.2 Elasticity Problems

Example 10.3 (Unit Elastic Demand)**.** Given $Q = 1000 \cdot P^{-1}$ (or equivalently, $P \cdot Q = 1000$):

This is a **unit elastic** demand curve because:

$$\varepsilon = \frac{dQ}{dP} \cdot \frac{P}{Q} = (-1000 \cdot P^{-2}) \cdot \frac{P}{1000 \cdot P^{-1}} = -1000 \cdot P^{-2} \cdot \frac{P^2}{1000} = -1$$

Properties of unit elastic demand:

- If $|\varepsilon| = 1$: $P \uparrow \Rightarrow Q \downarrow$ but TE unchanged
- Total expenditure $= P \times Q = 1000$ (constant)

Example 10.4 (General Cobb-Douglas)**.** Given $U = X^2Y^{1/2}$, $P_X = 100$, $P_Y = 50$, $I = 10,000$:

Marginal utilities:

$$MU_X = 2X \cdot Y^{1/2}$$
$$MU_Y = \frac{1}{2}X^2 \cdot Y^{-1/2}$$

Optimality:

$$\frac{MU_X}{MU_Y} = \frac{2XY^{1/2}}{\frac{1}{2}X^2Y^{-1/2}} = \frac{4Y}{X} = \frac{P_X}{P_Y} = \frac{100}{50} = 2$$

So $X = 2Y$.

Budget: $100X + 50Y = 10,000$

Substituting: $100(2Y) + 50Y = 250Y = 10,000 \Rightarrow Y = 40$, $X = 80$

Example 10.5 (Lump Sum Principle)**.** Consider a consumer with utility $U = X^2Y^{1/2}$, income $I = 10,000$, $P_X = 100$, $P_Y = 50$.

From the previous example, the optimal bundle is $X = 80$, $Y = 40$, giving:

$$U = 80^2 \times 40^{1/2} = 6400 \times 6.32 \approx 40,427$$

Scenario 1: Per-Unit Tax on X
Suppose we impose a per-unit tax that doubles P_X to 200.
New optimality: $\frac{4Y}{X} = \frac{200}{50} = 4 \Rightarrow X = Y$
Budget: $200X + 50X = 250X = 10,000 \Rightarrow X = Y = 40$
New utility: $U = 40^2 \times 40^{1/2} = 1600 \times 6.32 \approx 10,119$
Tax revenue: $(200 - 100) \times 40 = 4,000$
Scenario 2: Lump Sum Tax
Instead, impose a lump sum tax of $4,000 (same revenue).
New income: $10,000 - 4,000 = 6,000$
With original prices ($P_X = 100$, $P_Y = 50$), optimality gives $X = 2Y$:

$$100(2Y) + 50Y = 250Y = 6,000 \Rightarrow Y = 24, \quad X = 48$$

New utility: $U = 48^2 \times 24^{1/2} = 2304 \times 4.90 \approx 11,289$
Conclusion: The lump sum tax yields the same revenue but higher utility (11,289 vs 10,119).

Key Point

The **Lump Sum Principle**: A lump sum tax is more efficient than a per-unit tax that raises the same revenue because it does not distort the relative price ratio. Consumers can still optimize their purchasing choices based on true relative prices.

Part III

Producer Theory

11 Production

11.1 Introduction to Firm Behavior

We now turn from consumer theory to **producer theory**, examining how firms make decisions. The key topics include:

1. **Demand for inputs**: How firms decide how much labor, capital, and other inputs to use
2. **Input substitution**: How firms trade off between different inputs
3. **Technological change**: How production possibilities change over time

11.2 The Production Function

Definition 11.1 (Production Function)**.** A **production function** describes the relationship between inputs and outputs:

$$q = f(K, L)$$

where:

- q = quantity of output
- K = capital (machines, equipment, buildings)
- L = labor (workers, hours worked)

11.3 Marginal Product

Definition 11.2 (Marginal Product)**.** The **marginal product** of an input is the additional output that can be produced by adding one more unit of that input, holding all other inputs constant.

$$MP_L = \frac{\partial f(K, L)}{\partial L} \quad \text{(Marginal Product of Labor)}$$

$$MP_K = \frac{\partial f(K, L)}{\partial K} \quad \text{(Marginal Product of Capital)}$$

Note

The marginal product typically exhibits **diminishing returns**: as more of an input is used (holding others constant), each additional unit contributes less to output.

11.4 Average Product

Definition 11.3 (Average Product)**.** The **average product** is total output divided by total input:

$$AP_L = \frac{q}{L} = \frac{f(K, L)}{L}$$

For example, output per worker measures the average productivity of labor.

Note

Average product tells you the average productivity of all your workers, but it won't tell you how productive each individual worker is at the margin.

11.5 Isoquants

Definition 11.4 (Isoquant)**.** An **isoquant** shows the various combinations of inputs that produce the same amount of output. It is similar to an indifference curve but for production.

An **isoquant map** is a contour map of a firm's production function, showing multiple isoquants for different output levels.

Points on the same isoquant satisfy:

$$f(K_A, L_A) = f(K_B, L_B) = \bar{q}$$

11.6 Rate of Technical Substitution

Definition 11.5 (Rate of Technical Substitution (RTS))**.** The **rate of technical substitution** is the amount by which one input can be reduced when one more unit of another input is added, while holding output constant.

$$RTS = -\frac{\Delta K}{\Delta L} = -\text{slope of the isoquant}$$

Equivalently:

$$RTS = \frac{MP_L}{MP_K}$$

Note

The RTS answers the question: If you hire one more worker, how much less capital could you use and still produce the same level of output?

Property 11.1 (Diminishing RTS)**.** As a firm uses more and more labor:

- MP_L decreases (diminishing marginal product)

- Each additional worker leads to less saving on capital
- The RTS decreases along the isoquant (gets flatter)

This is why isoquants are convex to the origin.

11.7 Returns to Scale

Definition 11.6 (Returns to Scale)**. Returns to scale** describes the rate at which output increases in response to a proportional increase in all inputs.

Two key issues to consider:

- **Advantages of scale**: Greater division of labor, specialization
- **Disadvantages of scale**: Managerial inefficiencies, coordination problems

11.8 Constant Returns to Scale

Definition 11.7 (Constant Returns to Scale)**.** A production function exhibits **constant returns to scale** if multiplying all inputs by a factor n results in output increasing by the same factor n:

$$f(nK, nL) = n \cdot f(K, L)$$

For example, if $f(2K, 2L) = 2f(K, L)$, doubling inputs exactly doubles output.

11.9 Increasing Returns to Scale

Definition 11.8 (Increasing Returns to Scale)**.** A production function exhibits **increasing returns to scale** if multiplying all inputs by a factor n results in output increasing by more than n:

$$f(nK, nL) > n \cdot f(K, L)$$

Output increases at a rate greater than the increase in inputs. This often occurs due to specialization and division of labor.

11.10 Decreasing Returns to Scale

Definition 11.9 (Decreasing Returns to Scale)**.** A production function exhibits **decreasing returns to scale** if multiplying all inputs by a factor n results in output increasing by less than n:

$$f(nK, nL) < n \cdot f(K, L)$$

Output increases at a rate less than the increase in inputs. This often occurs due to coordination problems in large organizations.

11.11 Input Substitution

The degree of substitution between inputs varies across industries:

- **High substitutability**: Manufacturing (can substitute machines for workers)
- **Low substitutability**: Some services require specific input combinations

- **Fixed proportions**: Some technologies require inputs in fixed ratios (like 1 worker per lawnmower)

Example 11.1 (Fixed Proportions - Leontief Production)**.** In agriculture, comparing the U.S. and sub-Saharan Africa:

- U.S. agriculture: Capital-intensive (tractors, combines)
- Sub-Saharan Africa: Labor-intensive (manual farming)

The degree of substitution between capital and labor is a key economic issue affecting development paths.

11.12 Technological Change

Definition 11.10 (Technological Progress)**. Technological progress** shifts the firm's entire isoquant map inward, allowing the same output to be produced with fewer inputs.

Types of technological change:

- **Labor-saving**: Shifts isoquants to save on labor (e.g., automation)
- **Capital-saving**: Shifts isoquants to save on capital
- **Neutral**: Reduces both inputs proportionally

Example 11.2 (Production Function Calculations)**.** Given: $q = 2\sqrt{K \times L}$

Marginal Products:

$$MP_L = \frac{\partial q}{\partial L} = 2 \cdot \frac{1}{2} \cdot K^{1/2} \cdot L^{-1/2} = \sqrt{\frac{K}{L}}$$

$$MP_K = \frac{\partial q}{\partial K} = 2 \cdot \frac{1}{2} \cdot K^{-1/2} \cdot L^{1/2} = \sqrt{\frac{L}{K}}$$

Rate of Technical Substitution:

$$RTS = \frac{MP_L}{MP_K} = \frac{\sqrt{K/L}}{\sqrt{L/K}} = \frac{K}{L}$$

If $K = 4L$ and $q = 100$:

$$100 = 2\sqrt{4L \times L} = 2\sqrt{4L^2} = 4L \Rightarrow L = 25, \quad K = 100$$

11.13 Recap: Production Concepts

Key Point

Key production concepts:

- **Production function** $q = f(K, L)$: Relationship between inputs and output
- **Isoquant map**: Shows relationship between inputs and output (declining isoquants for higher output)
- **Marginal product**: Extra output produced by +1 input (holding other inputs constant)
- **Slope of isoquant** = Rate of Technical Substitution (RTS)
- **RTS** = Rate of substitution between two inputs, holding output constant
- RTS decreases as more of input X is used
- **Returns to scale**: How does output respond to proportional changes in all inputs?
- **Technological progress**: Shifts the firm's entire isoquant map

12 Costs

12.1 Introduction

The firm's cost problem involves two key questions:

1. How to choose inputs to produce a given amount of output at the lowest possible cost?
2. How do costs change as the firm changes the amount and mix of its inputs?

12.2 Basic Cost Concepts

Definition 12.1 (Opportunity Cost)**. Opportunity cost** is measured by the alternative uses that are foregone by producing a good. This includes the value of the owner's time.

Definition 12.2 (Accounting Cost)**. Accounting cost** is the actual cost paid for inputs—the explicit monetary outlays.

Definition 12.3 (Economic Cost)**. Economic cost** is the value of all inputs in their best alternative use. It includes both explicit and implicit costs.

12.3 Types of Costs

- **Explicit costs**: Cash outlays for inputs
  - Wages (labor costs)
  - Capital costs (rental rate)
- **Implicit costs**: Opportunity costs of owned resources

  - **Entrepreneurial costs**: Salary that the owner of the firm could earn at her next best employment

Definition 12.4 (Economic Profit)**. Economic profit** is revenue minus total cost (including both implicit and explicit costs):

$$\pi_{economic} = \text{Revenue} - \text{Total Economic Cost}$$

12.4 The Simple Cost Model

Assumptions:

- Two inputs: Labor (L) and Capital (K)
- Input markets are competitive
- w = wage rate (price of labor)
- v = rental rate (price of capital)

12.5 Total Cost

$$TC = wL + vK$$

12.6 Profit Function

$$\pi = Pq - TC$$
$$\pi = P \cdot f(K, L) - wL - vK$$

where:

- π depends on K and L
- P, w, v are given (market prices)

12.7 Cost Minimization

Definition 12.5 (Cost-Minimizing Equilibrium)**.** The firm's objective is to **minimize the cost** of producing a given output level. At the cost-minimizing equilibrium:

$$RTS = \frac{w}{v}$$

This is the least-cost option for the firm.

Note

The condition $RTS = w/v$ means that the amount of capital saved by employing one more worker equals the ratio of input prices.

12.8 Isocost Lines

Definition 12.6 (Isocost Line)**.** An **isocost line** shows all combinations of inputs that have the same total cost:

$$TC = wL + vK$$

Rearranging: $K = \frac{TC}{v} - \frac{w}{v}L$
The slope of the isocost line is $-\frac{w}{v}$.

Example 12.1 (Cost Minimization)**.** Given: $q = 200$, $w = 4$, $v = 1$
Total cost: $TC = wL + vK = 4L + K$
If we use only capital: $TC = 4 \times 100 + 1 \times 100 = 500$
The isocost line:

- If $L = 0$: $K = TC/v = TC/1 = TC$
- If $K = 0$: $L = TC/w = TC/4$

Slope of isocost $= \frac{TC/v}{TC/w} = \frac{w}{v} = 4$
At the cost-minimizing point, the isocost line is tangent to the isoquant:

$$\text{Rate of Technical Substitution} = RTS = \frac{MP_L}{MP_K} = \frac{w}{v}$$

Example 12.2 (Cost Minimization with Cobb-Douglas)**.** Given: $q = 2\sqrt{KL}$, produce $q = 200$, $w = 4$, $v = 1$
Step 1: Find marginal products

$$MP_L = \frac{\partial f(K,L)}{\partial L} = 2 \cdot \frac{1}{2} \cdot K^{1/2} \cdot L^{-1/2} = K^{1/2}L^{-1/2}$$

$$MP_K = \frac{\partial f(K,L)}{\partial K} = 2 \cdot \frac{1}{2} \cdot K^{-1/2} \cdot L^{1/2} = K^{-1/2}L^{1/2}$$

Step 2: Find RTS

$$\frac{MP_L}{MP_K} = \frac{K^{1/2}L^{-1/2}}{K^{-1/2}L^{1/2}} = \frac{K}{L}$$

Step 3: Set RTS = price ratio

$$\frac{MP_L}{MP_K} = \frac{K}{L} = \frac{w}{v} = \frac{4}{1} = 4$$

So at equilibrium: $K = 4L$
Step 4: Plug into production function

$$f(K,L) = 200 = 2\sqrt{K \times L}$$

$$100 = \sqrt{KL} = \sqrt{4L \times L} = 2L$$

$$L = 50, \quad K = 200$$

Step 5: Calculate total cost

$$TC = wL + vK = 4(50) + 1(200) = 200 + 200 = \$400$$

12.9 Total Cost and Returns to Scale

The shape of the total cost curve depends on the returns to scale:

- **Constant Returns to Scale**: Production and cost expand at the same rate. The TC curve is linear.
- **Decreasing Returns to Scale**: Costs expand more rapidly than output. The TC curve is convex (curves upward).
- **Increasing Returns to Scale**: Costs expand less rapidly than output. The TC curve is concave (curves downward).

Example 12.3 (Verifying Constant Returns to Scale)**.** Given: $q = 2\sqrt{K \times L}$
If we double inputs:

$$q_2 = 2\sqrt{2K \times 2L} = 2\sqrt{4KL} = 4\sqrt{KL} = 2q_1$$

So $f(2K, 2L) = 2f(K, L)$—this exhibits **constant returns to scale**.
If $q = 200$ requires $L = 50$, $K = 200$, then $q = 400$ requires $L = 100$, $K = 400$.
With $w = 4$, $v = 1$:

- $TC(200) = 4(50) + 1(200) = 400$
- $TC(400) = 4(100) + 1(400) = 800$

Doubling output exactly doubles cost—confirming constant returns to scale.

12.10 Average and Marginal Cost

Definition 12.7 (Average Cost)**. Average cost** (AC) is total cost divided by output:

$$AC = \frac{TC}{q}$$

It represents the cost per unit of output.

Definition 12.8 (Marginal Cost)**. Marginal cost** (MC) is the cost of producing one additional unit:

$$MC = \frac{dTC}{dq} = \frac{\Delta TC}{\Delta q}$$

12.11 Relationship Between AC and MC

Property 12.1 (AC and MC Relationship)**.** The relationship between average cost and marginal cost depends on returns to scale:

- **Constant returns to scale**: $AC = MC$ (both are constant)
- **Decreasing returns to scale**: $MC > AC$, and both are increasing
- **Increasing returns to scale**: $MC < AC$, and both are decreasing

Key Point

When $MC < AC$, average cost is falling (each additional unit costs less than average).
When $MC > AC$, average cost is rising (each additional unit costs more than average).
When $MC = AC$, average cost is at its minimum. This output level is called the **efficient scale**.

12.12 Short-Run vs. Long-Run Costs

Definition 12.9 (Short Run vs. Long Run)**.**

- **Short run**: Some inputs are fixed (cannot be adjusted)
- **Long run**: All inputs are variable (can be adjusted)

Definition 12.10 (Fixed and Variable Costs)**.**

- **Fixed costs**: Costs associated with inputs that are fixed in the short run (e.g., capital, rent)
- **Variable costs**: Costs associated with inputs that can be varied in the short run (e.g., labor, materials)

In the production function $f(K, L)$:

- In the short run: K is fixed at $\bar{K}$, only L is variable
- This introduces inflexibility—the firm cannot achieve the optimal input mix

Note

In the short run, the cost-minimization condition $RTS = w/v$ will generally **not hold** because the firm cannot adjust capital. The firm must use more labor than optimal to compensate for fixed capital.

12.13 Shifts in Cost Curves

Cost curves can shift due to:

1. **Changes in input prices**: If $w \uparrow$, how do K^* and L^* change? Depends on the firm's capacity for substitution.
2. **Technological innovation**: Cost curves shift down. Tech change might be biased (labor-saving or capital-saving).
3. **Economies of scope**: In multi-product firms, expansion of one product may improve the ability to produce another.

12.14 Short-Run Cost Analysis: Worked Example

Example 12.4 (Short-Run Cost Functions)**.** Given: $q = 2\sqrt{K \times L}$, $\bar{K} = 100$ (fixed), $v = \$1$, $w = \$4$

Part (a): Derive Short-Run Cost Functions

In the short run with $K = 100$:

$$q = 2\sqrt{100 \cdot L} = 20\sqrt{L}$$

Solving for L:

$$\sqrt{L} = \frac{q}{20} \Rightarrow L = \frac{q^2}{400}$$

Short-run total cost:

$$STC = wL + vK = 4 \cdot \frac{q^2}{400} + 1 \cdot 100 = \frac{q^2}{100} + 100$$

Short-run average cost:

$$SATC = \frac{STC}{q} = \frac{q}{100} + \frac{100}{q}$$

Short-run marginal cost:

$$SMC = \frac{dSTC}{dq} = \frac{2q}{100} = \frac{q}{50}$$

Part (b): Calculate Costs at Various Output Levels
At $q = 25$:

$$STC = \frac{625}{100} + 100 = 106.25$$
$$SATC = \frac{25}{100} + \frac{100}{25} = 0.25 + 4 = 4.25$$
$$SMC = \frac{25}{50} = 0.50$$

At $q = 50$:

$$STC = \frac{2500}{100} + 100 = 125$$
$$SATC = \frac{50}{100} + \frac{100}{50} = 0.5 + 2 = 2.50$$
$$SMC = \frac{50}{50} = 1.00$$

At $q = 100$ (Efficient Scale):

$$STC = \frac{10000}{100} + 100 = 200$$
$$SATC = \frac{100}{100} + \frac{100}{100} = 1 + 1 = 2.00$$
$$SMC = \frac{100}{50} = 2.00$$

Note: At $q = 100$, $SATC = SMC = 2$. This is the efficient scale where average cost is minimized.

To verify: Set $SATC = SMC$:

$$\frac{q}{100} + \frac{100}{q} = \frac{q}{50}$$

$$\frac{100}{q} = \frac{q}{100} \Rightarrow q^2 = 10,000 \Rightarrow q^* = 100$$

At $q = 200$:

$$SATC = \frac{200}{100} + \frac{100}{200} = 2 + 0.5 = 2.50$$
$$SMC = \frac{200}{50} = 4.00$$

Part (c): Long-Run Optimal Input Mix for $q = 200$
In the long run, the firm can adjust both K and L.
From earlier: $q = K^{1/2}L^{1/2}$ (rewriting $q = 2\sqrt{KL}$ as $q/2 = \sqrt{KL}$)
Marginal products:

$$MP_L = \frac{1}{2}K^{1/2}L^{-1/2}, \quad MP_K = \frac{1}{2}K^{-1/2}L^{1/2}$$

RTS:
$$\frac{MP_L}{MP_K} = \frac{K}{L} = \frac{w}{v} = \frac{4}{1} = 4 \Rightarrow K = 4L$$

Substituting into the production function for $q = 200$:

$$200 = 2\sqrt{KL} = 2\sqrt{4L \cdot L} = 4L \Rightarrow L = 50, \quad K = 200$$

Long-run total cost: $TC = 4(50) + 1(200) = 400$

13 Profit Maximization and Supply

13.1 Introduction

We now connect production and costs to the firm's ultimate objective: **profit maximization**.
Key concepts:

- **Production**: Relationship between inputs and outputs
- **Costs**: Relationship between output and costs
- **Output decision**: How much to produce to maximize profit

13.2 The Profit Function

Definition 13.1 (Profit)**. Profit** (π) is total revenue minus total cost:

$$\pi = TR - TC$$

Economic profit includes **explicit costs** (wages, capital costs) and **implicit costs** (opportunity costs).

Property 13.1 (Rational Firm Behavior)**.** Firms are rational and think at the margin (incrementally):

- Does my profit increase if I produce one more unit?
- If yes, increase quantity ($q \uparrow$)
- If no, decrease quantity ($q \downarrow$)

13.3 Profit Maximization Condition

Definition 13.2 (Profit Maximization)**.** To maximize profit, choose q such that:

$$\pi(q) = TR(q) - TC(q)$$

Taking the first-order condition:

$$\frac{d\pi}{dq} = 0 \Rightarrow \frac{dTR(q)}{dq} - \frac{dTC(q)}{dq} = 0$$

$$MR - MC = 0 \Rightarrow \boxed{MR = MC}$$

Key Point

The profit-maximizing condition is $MR = MC$:

- **MR** = Marginal Revenue = Extra revenue from selling +1 unit
- **MC** = Marginal Cost = Cost of producing that extra unit

Why?

- If $MR > MC$: Produce more ($q \uparrow$) $\Rightarrow \pi \uparrow$
- If $MR < MC$: Produce less ($q \downarrow$) $\Rightarrow \pi \uparrow$
- In either case, π is not maximized until $MR = MC$

Example 13.1 (Profit Maximization)**.** Given: Price $P = \$20$, Total cost $TC = 50 + 10q + 0.1q^2$

1. Total Revenue:

$$TR = P \times q = 20q$$

$$MR = \frac{dTR}{dq} = 20$$

2. Cost Analysis:

- Fixed cost = 50
- Variable cost = $10q + 0.1q^2$
- Average cost: $AC = \frac{50}{q} + 10 + 0.1q$
- Marginal cost: $MC = \frac{dTC}{dq} = 10 + 0.2q$

3. Profit Maximization:

$$\pi = TR - TC = 20q - (50 + 10q + 0.1q^2)$$

Setting $\frac{d\pi}{dq} = 0$:

$$MR = MC \Rightarrow 20 = 10 + 0.2q$$

$$0.2q = 10 \Rightarrow q^* = 50$$

4. Maximum Profit:

$$TR = 20 \times 50 = 1,000$$

$$TC = 50 + 10(50) + 0.1(50)^2 = 50 + 500 + 250 = 800$$

$$\pi^* = 1,000 - 800 = \$200$$

13.4 Price Takers vs. Price Makers

13.4.1 Price Taker (Perfect Competition)

Definition 13.3 (Price Taker)**.** A **price taker** is a firm that cannot influence the market price. It sells all units at the same market price P.

$$MR = P$$

The marginal revenue equals the price for every unit sold.

13.4.2 Price Maker (Monopoly)

Definition 13.4 (Price Maker)**.** A **price maker** is a firm that can influence the market price. The most extreme case is a monopoly—the only firm in the market that individually faces the market demand.

To sell more, the monopolist must lower the price for **all** units.

Example 13.2 (Monopolist's Marginal Revenue)**.** Suppose demand is $q_D = 10 - P$, so $P = 10 - q$.

q	P	$TR = P \times q$
1	9	9
2	8	16
3	7	21
4	6	24
5	5	25
6	4	24
7	3	21
8	2	16
9	1	9
10	0	0

When quantity increases from 3 to 4:

- **Gain**: Sell 1 more unit at \$6 $\Rightarrow$ +\$6
- **Loss**: Price drops from \$7 to \$6 on first 3 units $\Rightarrow$ −\$3
- **Net MR** $= 6 - 3 = 3$

The marginal revenue is less than the price because the firm must lower the price on all units to sell one more.

13.5 Marginal Revenue and Elasticity

Definition 13.5 (Price Elasticity of Demand)**.** The **price elasticity of demand** measures responsiveness of quantity to price:

$$\varepsilon_{q,P} = \frac{\%\Delta Q}{\%\Delta P} = \frac{dQ/Q}{dP/P}$$

For a firm: $\varepsilon_{q,P} = \frac{\%\Delta q}{\%\Delta P}$ (firm-specific demand elasticity)

Theorem 13.1 (MR-Elasticity Relationship)**.** Marginal revenue is related to price and elasticity by:

$$MR = P\left(1 + \frac{1}{\varepsilon_{q,P}}\right)$$

Since demand curves are downward sloping, $\varepsilon_{q,P} < 0$.

Proof. Total revenue: $TR = P(q) \cdot q$

Marginal revenue:

$$MR = \frac{dTR}{dq} = \frac{d[P(q) \cdot q]}{dq} = \frac{dP}{dq} \cdot q + P \cdot 1$$

$$MR = P\left(\frac{dP}{dq} \cdot \frac{q}{P} + 1\right) = P\left(\frac{dP/P}{dq/q} + 1\right) = P\left(\frac{1}{\varepsilon_{q,P}} + 1\right)$$

□

Key Point

Implications of the MR-Elasticity relationship:

- If $|\varepsilon| > 1$ (elastic): $MR > 0$, so $P \downarrow \Rightarrow TR \uparrow$
- If $|\varepsilon| = 1$ (unit elastic): $MR = 0$, TR is at maximum
- If $|\varepsilon| < 1$ (inelastic): $MR < 0$, so $P \downarrow \Rightarrow TR \downarrow$

A profit-maximizing monopolist will never operate in the inelastic portion of demand (where $MR < 0$), because they could increase profit by raising price and reducing quantity.

13.5.1 MR and Elasticity: Special Cases

Property 13.2 (Elasticity and MR)**.** From the formula $MR = P\left(1 + \frac{1}{\varepsilon_{q,P}}\right)$:

- If $\varepsilon_{q,P} = -\infty$ (perfectly elastic): $MR = P$ — this is **perfect competition**
- If $\varepsilon_{q,P} < -1$ (elastic): $MR > 0$
- If $\varepsilon_{q,P} = -1$ (unit elastic): $MR = 0$
- If $-1 < \varepsilon_{q,P} < 0$ (inelastic): $MR < 0$

Since $\varepsilon_{q,P} < 0$ (demand curves slope downward), we always have $P > MR$.

13.5.2 MR Curve for Linear Demand

Property 13.3 (Linear Demand and MR)**.** If the inverse demand curve is linear: $P = a - bq$

Then:

$$TR = P \times q = (a - bq)q = aq - bq^2$$

$$MR = \frac{dTR}{dq} = a - 2bq$$

Key result: The MR curve has the same intercept as the demand curve but **twice the slope**. If $P = a - bq$, then $MR = a - 2bq$.

Example 13.3 (Linear Demand MR Curve)**.** If $P = 10 - q$ (so $a = 10$, $b = 1$):

$$TR = (10 - q)q = 10q - q^2$$
$$MR = \frac{dTR}{dq} = 10 - 2q$$

The demand curve intersects the quantity axis at $q = 10$ (where $P = 0$).
The MR curve intersects the quantity axis at $q = 5$ (where $MR = 0$).

Part IV

Market Structures

14 Perfect Competition

14.1 Introduction

Key questions in perfect competition:

- How is the price determined? $\rightarrow$ Production decisions
- Market entry and exit
- Welfare implications

14.2 Time Frames for Production Decisions

Production decisions depend on the time frame:

1. **Very Short Run**: Quantity is fixed
   - No entry/exit
   - No change in supply
   - Market can't respond to demand changes
2. **Short Run**: Existing firms can change quantity
   - q for existing firms can change
   - But no entry/exit
3. **Long Run**: Full flexibility
   - Both q from existing firms can change
   - Firms can also enter and exit the industry

14.3 Very Short-Run Supply

In the very short run, supply is **perfectly inelastic** (vertical supply curve). Quantity is fixed at $\bar{Q}$.

When demand shifts:

- If demand increases ($D \rightarrow D'$): Price rises but quantity stays at $\bar{Q}$
- Price serves as a **rationing mechanism**—only consumers with highest willingness to pay get the product

14.4 Short-Run Supply

In the short run:

- Firms can change quantity q
- No entry or exit
- Use individual supply curves for each firm and aggregate

14.4.1 Individual and Market Supply

The **market supply curve** is the horizontal sum of all individual firm supply curves:

$$Q_S = q_A + q_B + q_C + \dots$$

Note

The market supply curve is **more elastic** than individual firm supply curves because there are many firms (potentially thousands) whose individual responses aggregate.

14.4.2 Short-Run Equilibrium for a Typical Firm

For a price-taking firm in perfect competition:

- The firm faces a horizontal demand curve at the market price P
- $MR = P$ (since the firm can sell any quantity at price P)
- Profit maximization: $MR = P = MC$
- The profit-maximizing quantity q^* is where $P = MC$

Definition 14.1 (Efficient Scale)**.** The **efficient scale** $\bar{q}$ is the output level where average cost is minimized (where $MC = AC$).

14.5 Short-Run Response to Demand Shocks

Positive demand shock: Demand shifts right ($D \to D'$)

Market response:

- Price rises from P_1 to P_2
- Quantity increases from Q_1 to Q_2
- The additional quantity comes from existing firms expanding production

Firm response:

- Price signal tells firms to produce more
- Each firm moves up its MC curve
- Profit: $\pi = (P - ATC) \times q$
- If $P > ATC$: positive profit
- If $P < ATC$: negative profit (loss)
- If $P = ATC$: zero economic profit

Note

In the short run, firms can operate with $\pi > 0$, $\pi < 0$, or $\pi = 0$, given the market price. Price serves two purposes:

1. **Signal to producers**: $P = SMC$ determines how much to produce
2. **Rations demand**: Consumers with higher willingness to pay buy the product

Example 14.1 (Short-Run Profit Calculation)**.** Given: $STC = 0.1q^2 + 10q + 250$

Fixed cost: $FC = 250$

Variable cost: $VC = 0.1q^2 + 10q$

Average variable cost: $AVC = 0.1q + 10$

Average total cost: $ATC = 0.1q + 10 + \frac{250}{q}$

Marginal cost: $MC = 0.2q + 10$

Finding the efficient scale (where $MC = ATC$):

$$0.2q + 10 = 0.1q + 10 + \frac{250}{q}$$

$$0.1q = \frac{250}{q} \Rightarrow 0.1q^2 = 250 \Rightarrow q^2 = 2500 \Rightarrow q^* = 50$$

At $q = 50$: $ATC = 0.1(50) + 10 + \frac{250}{50} = 5 + 10 + 5 = 20$

So the efficient scale is $q^* = 50$ with $ATC = \$20$.

If market price $P = 18$:

Profit-maximizing output: $MC = P$

$$0.2q + 10 = 18 \Rightarrow q^{**} = 40$$

At $q = 40$: $ATC = 0.1(40) + 10 + \frac{250}{40} = 4 + 10 + 6.25 = 20.25$

Profit:

$$\pi = (P - ATC) \times q = (18 - 20.25) \times 40 = -2.25 \times 40 = -\$90$$

Should the firm operate?

Compare operating vs. shutting down:

- If $q = 0$: $\pi = 0 - 250 = -\$250$ (lose all fixed costs)
- If $q = 40$: $\pi = -\$90$

Operating is better! The firm should continue because $P > AVC$ (price covers variable costs and contributes to fixed costs).

Shutdown rule: Operate if $P \geq AVC$; shut down if $P < AVC$.

14.6 Types of Demand and Supply Shocks

14.6.1 Demand Shocks

Positive demand shocks (demand shifts right):

- Increase in income (for normal goods)
- Increase in price of a substitute
- Decrease in price of a complement
- Increase in preferences for the good

14.6.2 Supply Shocks

Positive supply shocks (supply shifts right):

- Decrease in input prices
- Improvement in technology

14.6.3 Elasticity and Equilibrium Changes

The change in equilibrium price and quantity depends on the price elasticity of demand and supply.

Property 14.1 (Supply Shock with Different Demand Elasticities)**.** For a given supply shock:

- With **elastic demand**: Large quantity change, small price change
- With **inelastic demand**: Small quantity change, large price change

Property 14.2 (Demand Shock with Different Supply Elasticities)**.** For a given demand shock:

- With **elastic supply**: Large quantity change, small price change
- With **inelastic supply**: Small quantity change, large price change

14.7 Long-Run Supply

In the long run:

- Supply responses are more flexible
- Firms have greater input flexibility
- Firms can enter and exit the industry

14.7.1 Long-Run Equilibrium Conditions

Definition 14.2 (Long-Run Competitive Equilibrium)**.** In long-run equilibrium:

1. Entry and exit stop: $\pi = 0$ (zero economic profit)
2. All firms maximize profit: $MC = P$

Combining these: In long-run equilibrium, $P = MC = AC$ (price equals marginal cost equals average cost).

14.7.2 Entry and Exit Dynamics

If $\pi > 0$ (positive economic profit):

- Firms will enter the industry
- Opportunity to do better than best outside option
- Supply shifts right ($S \to S'$)
- Price falls, quantity increases

- Process continues until $\pi = 0$

If $\pi < 0$ (negative economic profit):

- Firms will exit the industry
- Better opportunities exist elsewhere
- Supply shifts left ($S' \to S$)
- Price rises, quantity decreases
- Process continues until $\pi = 0$

Key Point

In long-run competitive equilibrium:

- $MC = P$ (profit maximization)
- $\pi = 0$ (zero economic profit, so $P = AC$)
- Combined: $P = MC = AC$
- Firms operate at efficient scale (minimum AC)

"Doing as good as your best outside option" means zero **economic** profit, not zero accounting profit.

14.7.3 Graphical Analysis of Long-Run Equilibrium

In long-run equilibrium, firms in perfect competition produce at the **efficient scale**—the output level where average total cost is minimized.

The equilibrium condition $P = AC = MC$ can be derived as follows:

- Firm entry/exit will stop when $\pi = 0$
- Profit: $\pi = P \times q - TC = P \times q - ATC \times q = q(P - ATC)$
- Given $q > 0$, $\pi = 0$ implies $P = ATC$
- But also $MC = P$ (profit maximization)
- Combined: $P = AC = MC$

At this point, MC intersects AC at its minimum, so firms produce at the efficient scale.

14.7.4 Long-Run Response to Demand Shocks

Consider a positive demand shock ($D \to D'$):

Short-run response:

- Price rises from P_1 to P_2
- Quantity increases from Q_1 to Q_2

- Existing firms earn positive profit: $\pi = q \times (P_2 - ATC) > 0$

Long-run response:

- $\pi > 0$ attracts new firms to enter
- Supply shifts right as more firms enter
- Price falls back toward original level
- Total quantity increases further to Q_3
- Process continues until $\pi = 0$

Note

The long-run adjustment process is quite mechanical—it does not assume any interaction between ATC and the number of firms. However, with network externalities or knowledge spillovers, more firms could lead to lower ATC (as firms share knowledge), resulting in a downward-sloping long-run supply curve.

14.8 Consumer and Producer Surplus

Definition 14.3 (Consumer Surplus)**. Consumer surplus (CS)** is the extra value that consumers receive over what they pay for a good:

$$CS = \text{Willingness to Pay} - \text{Price Paid}$$

Graphically, consumer surplus is the area below the demand curve and above the market price.

Definition 14.4 (Producer Surplus)**. Producer surplus (PS)** is the extra value that producers receive for a good in excess of the opportunity cost of production:

$$PS = \text{Price Received} - \text{Opportunity Cost of Production}$$

We assume that the supply curve represents the opportunity cost. Graphically, producer surplus is the area above the supply curve and below the market price.

Marginal analysis of surplus:

- Any consumer with $WTP \geq P$ will buy (and receive surplus)
- The last consumer to buy has $WTP = P$ (zero surplus)
- All other consumers with $WTP > P$ receive positive surplus
- All producers with Cost $\leq P$ will sell (and receive surplus)
- The last producer to join has Cost $= P$ (zero surplus)
- All other producers with Cost $< P$ receive positive surplus

Key Point

The single price that applies to all firms and consumers means that:

- Consumers with high willingness to pay get surplus
- Low-cost producers get surplus
- The marginal buyer and marginal seller earn zero surplus

14.9 Ricardian Rent

Definition 14.5 (Ricardian Rent)**. Ricardian rent** is the long-run profit of firms with low costs. This concept relaxes the assumption that all firms have identical cost structures.

With heterogeneous firms (different production technologies or cost structures):

- **Low-cost firm**: $\pi > 0$ (earns positive economic profit)
- **Medium-cost firm**: $\pi \geq 0$ (may earn some profit, $P = MC$ but $P > AC$)
- **Marginal firm**: $\pi = 0$ (last firm to enter, $P = MC = AC$)

The marginal firm (the last firm to enter the market) earns zero economic profit. All other firms with lower costs can still earn positive economic profit even in long-run equilibrium—this is Ricardian rent.

Note

Markets can still be perfectly competitive even when some firms earn positive profits due to cost advantages. The key is that the marginal firm earns zero profit, which determines when entry stops.

14.10 Economic Efficiency

Definition 14.6 (Economic Efficiency)**. Economic efficiency** occurs when the sum of consumer and producer surplus is maximized. It reflects the best use of society's resources.

Property 14.3 (Perfect Competition and Efficiency)**.** In perfect competition equilibrium, there are no more mutually beneficial exchanges possible. This efficiency result **only holds in perfectly competitive markets**.

At the competitive equilibrium:

- The supply curve represents the willingness to supply (minimum acceptable price) for the last firm to enter—the opportunity cost of resources to the market
- The demand curve represents the willingness to pay for the last consumer to buy—the value to consumers
- At equilibrium, these are equal for the marginal unit

Inefficiency from overproduction: If quantity exceeds Q^*, the cost of production to society exceeds the value to consumers for those additional units—a waste of resources.

Inefficiency from underproduction: If quantity is below Q^*, there are mutually beneficial exchanges not being realized ($Q^* - Q_1$ units where value exceeds cost).

14.11 Tax Incidence

Definition 14.7 (Tax Incidence)**. Tax incidence** refers to who bears the economic burden of a tax. Taxes reduce overall welfare by distorting the price mechanism.

Key Point

The burden of the tax does **not** depend on who has to legally pay it. The economic burden of the tax depends on the **price elasticity of supply and demand**.

14.11.1 Tax Analysis Framework

Assume firms legally pay the tax (the analysis is symmetric if buyers pay).

Effect of a per-unit tax t:

- Creates a wedge between buyer price (P_B) and seller price (P_S)
- $P_B = P_S + t$ (buyers pay more than sellers receive)
- Supply effectively shifts up by the amount of the tax
- Equilibrium quantity falls
- Deadweight loss (DWL) is created

14.11.2 Elasticity and Tax Burden

The relative size of consumer versus producer burden depends on elasticities:

Property 14.4 (Tax Burden and Elasticity)**.**

- If demand is **more elastic** than supply: Firms bear a higher share of the tax burden (sellers pay most of the tax)
- If supply is **more elastic** than demand: Consumers bear a higher share of the tax burden (buyers pay most of the tax)

The more inelastic side of the market bears more of the tax burden because they cannot easily adjust their behavior.

14.11.3 Deadweight Loss

Taxes reduce quantity traded and create **deadweight loss (DWL)**—the loss in total surplus that is not captured by anyone (not by consumers, producers, or the government).

Example 14.2 (Tax Incidence Calculation)**.** Given:

$$Q_S = 240P_S - 480$$
$$Q_D = 640 - 80P_B$$

Step 1: Find pre-tax equilibrium
Set $Q_S = Q_D$ with $P_S = P_B = P$:

$$240P - 480 = 640 - 80P$$
$$320P = 1120$$
$$P^* = \$3.50$$

Quantity: $Q^* = 240(3.5) - 480 = 840 - 480 = 360$
Step 2: Calculate pre-tax surplus
Consumer surplus: $CS = \frac{1}{2} \times (8 - 3.5) \times 360 = \frac{1}{2} \times 4.5 \times 360 = \$810$
Producer surplus: $PS = \frac{1}{2} \times (3.5 - 2) \times 360 = \frac{1}{2} \times 1.5 \times 360 = \$270$
Total surplus: $TS = CS + PS = 810 + 270 = \$1080$

Example 14.3 (Tax Incidence Calculation (continued))**.** Now impose a $2 per-unit tax on sellers.
Step 3: Find post-tax equilibrium
With tax: $P_B = P_S + 2$
Substitute into demand: $Q_D = 640 - 80(P_S + 2) = 640 - 80P_S - 160 = 480 - 80P_S$
Set $Q_S = Q_D$:

$$\begin{aligned} 240P_S - 480 &= 480 - 80P_S \\ 320P_S &= 960 \\ P_S &= \$3 \end{aligned}$$

Therefore: $P_B = 3 + 2 = \$5$
Quantity: $Q = 240(3) - 480 = 240$
Step 4: Calculate post-tax surplus
New consumer surplus: $CS' = \frac{1}{2} \times (8 - 5) \times 240 = \frac{1}{2} \times 3 \times 240 = \$360$
New producer surplus: $PS' = \frac{1}{2} \times (3 - 2) \times 240 = \frac{1}{2} \times 1 \times 240 = \$120$
Tax revenue: $T = 2 \times 240 = \$480$
Total welfare: $CS' + PS' + T = 360 + 120 + 480 = \$960$
Deadweight loss: $DWL = 1080 - 960 = \$120$
Analysis of tax burden:

- Price for buyers rose from $3.50 to $5.00 (increase of $1.50)
- Price for sellers fell from $3.50 to $3.00 (decrease of $0.50)
- Buyers bear 75% of the $2 tax ($1.50/$2)
- Sellers bear 25% of the $2 tax ($0.50/$2)

This is because demand is less elastic than supply—buyers cannot easily reduce their consumption, so they bear more of the burden.

Example 14.4 (Tax Burden Breakdown)**.** From the previous example, we can decompose the tax burden:
Tax revenue breakdown:

- Total tax revenue: $T = B + C = \$480$
- Buyers' share: $(5 - 3.5) \times 240 = 1.5 \times 240 = \$360$
- Firms' share: $(3.5 - 3) \times 240 = 0.5 \times 240 = \$120$

Deadweight loss calculation:

$$DWL = E + F = \frac{1}{2} \times (5 - 3) \times (360 - 240) = \frac{1}{2} \times 2 \times 120 = \$120$$

This represents the value of mutually beneficial trades that no longer occur due to the tax.

15 Monopoly

15.1 Introduction: Perfect Competition vs. Monopoly

Perfect Competition (review):

- Firms are **price takers** (no market power)
- Profit maximization: $P = MR = MC$
- Long-run equilibrium: $P = MR = MC = AC$, so $\pi = 0$
- Market is efficient (Pareto optimal)—overall surplus is maximized

Monopoly:

- Single firm with lots of **market power**
- Firm is a **price maker** (price setter)
- Uses pricing strategies to capture consumer surplus
- Creates deadweight loss

15.2 How Do Monopolies Emerge?

Monopolies arise from **barriers to entry**:

15.2.1 Legal Barriers

- **Patents and copyrights**: Government-induced monopoly to foster innovation
- **Exclusive franchise or license**: Government grants exclusive right to operate

15.2.2 Technical Barriers

- **Decreasing average cost**: Huge infrastructure investments create natural monopolies (e.g., utility companies, software companies)
- **Special knowledge of production**: Proprietary technology or lowest-cost production methods
- **Ownership of a key resource**: Control of essential inputs (e.g., diamond mines, oil)
- **Unique talent for production**: Specialized capabilities (e.g., nuclear technology)

15.3 Profit Maximization for a Monopolist

15.3.1 The Monopolist's Problem

The monopolist chooses quantity Q to maximize profit. Key insight:

- Industry output = Firm output (monopoly output)
- The firm faces the **market demand curve**

- Demand is downward sloping: every time the monopolist wants to sell one more unit, it has to lower the price

Key Point

For a monopolist, $MR < P$ always.
Unlike perfect competition where $MR = P$, the monopolist's marginal revenue is less than price because lowering price to sell one more unit also lowers the price on all previous units.

15.3.2 Profit Maximization Condition

The monopolist maximizes profit where:

$$MR = MC$$

Then the monopolist charges the price from the **demand curve** at that quantity:

$$P > MR = MC$$

Note

There is **no well-defined supply curve** for a monopoly. In perfect competition, output depends only on the MC curve. For a monopoly, supply cannot be analyzed without taking into account the demand curve—the MR curve depends on demand, so output depends on both MC and demand.

15.3.3 Monopoly Has No Supply Curve

In perfect competition, we can derive a supply curve independent of demand. For a monopoly:

- Different demand curves lead to different MR curves
- Different MR curves lead to different optimal outputs
- For each price P, output depends on the shape of demand
- Therefore, supply is a function of demand—no independent supply curve exists

15.4 Monopoly Profit

Definition 15.1 (Monopoly Profit)**.** The monopolist's economic profit is:

$$\pi = TR - TC = P \times Q - AC \times Q = Q(P - AC)$$

Since $P > MC$ and typically $P > AC$, the monopolist earns positive economic profit: $\pi > 0$.

Graphically, monopoly profit is the rectangle with:

- Height: $P - AC$ (profit per unit)
- Width: Q^* (quantity produced)

15.5 Problems with Monopoly

Compared to a competitive market, monopoly creates several issues:

1. **Price too high**: $P_M > P_{PC}$
2. **Output too low**: $Q_M < Q_{PC}$ (allocative inefficiency)
3. **Positive economic profit**: $\pi > 0$ (no entry to compete away profits)
4. **Welfare transfer**: Surplus transfers from consumers to the monopolist ("fat cat syndrome")
5. **Deadweight loss**: Allocative inefficiency—mutually beneficial trades don't occur

15.5.1 Welfare Analysis of Monopoly

Compared to perfect competition (where $P = MC = AC$):
Under monopoly:

- Consumer surplus decreases (higher price, lower quantity)
- Producer surplus (profit) increases
- Part of former CS is transferred to the monopolist as profit
- Part of former total surplus is lost entirely (DWL)

Example 15.1 (Monopoly vs. Perfect Competition)**.** Given: $Q = 53-P$ (equivalently, $P = 53-Q$), and $MC = AC = 5$ (constant).
Step 1: Find monopoly equilibrium
Total revenue: $TR = P \times Q = (53 - Q) \times Q = 53Q - Q^2$
Marginal revenue: $MR = \frac{dTR}{dQ} = 53 - 2Q$
Set $MR = MC$:

$$\begin{aligned} 53 - 2Q &= 5 \\ 48 &= 2Q \\ Q^*_M &= 24 \end{aligned}$$

Price: $P^*_M = 53 - 24 = 29$
Profit: $\pi = Q \times (P - AC) = 24 \times (29 - 5) = 24 \times 24 = \$576$
Step 2: Find perfect competition equilibrium
Under perfect competition: $P = MC = AC = 5$
From demand: $Q_{PC} = 53 - 5 = 48$
Profit: $\pi_{PC} = 0$ (since $P = AC$)
Step 3: Calculate welfare
Under monopoly:

$$CS_M = \frac{1}{2} \times (53 - 29) \times 24 = \frac{1}{2} \times 24 \times 24 = \$288$$

Producer surplus under monopoly equals profit:

$$PS_M = \pi_M = \$576$$

Total surplus under monopoly: $TS_M = CS_M + PS_M = 288 + 576 = \$864$
Under perfect competition:

$$CS_{PC} = \frac{1}{2} \times (53 - 5) \times 48 = \frac{1}{2} \times 48 \times 48 = \$1152$$

$PS_{PC} = \pi = 0$ (since $P = AC$)
Total surplus under perfect competition: $TS_{PC} = \$1152$
Step 4: Calculate deadweight loss

$$DWL = TS_{PC} - TS_M = 1152 - 864 = \$288$$

Alternatively, using the triangle formula:

$$DWL = \frac{1}{2} \times (P_M - MC) \times (Q_{PC} - Q_M) = \frac{1}{2} \times (29 - 5) \times (48 - 24) = \frac{1}{2} \times 24 \times 24 = \$288$$

Key Point

Summary of monopoly problems compared to perfect competition:

1. Price too high ($P_M > P_{PC}$)
2. Output too low ($Q_M < Q_{PC}$)
3. Positive economic profit ($\pi > 0$)
4. Welfare transfer from consumers to producers
5. Deadweight loss (allocative inefficiency)

15.6 Price Discrimination

Definition 15.2 (Price Discrimination)**. Price discrimination** occurs when identical products are sold at different prices based on consumer willingness to pay. The goal is to capture more consumer surplus and increase profit.

In an ideal world, a monopolist would be happy to sell as much as it could if $P \geq MC$. The problem with single pricing:

- Setting $P = P_M$ (monopoly price) is too high
- Pushes consumers with lower willingness to pay out of the market
- Leaves potential surplus uncaptured

15.6.1 Perfect Price Discrimination (First-Degree)

Definition 15.3 (Perfect Price Discrimination)**.** Under **perfect price discrimination**, the monopoly charges each consumer a price exactly equal to their willingness to pay.

Key features:

- Each consumer pays $P = WTP$, so consumer surplus $= 0$

- All surplus is transferred to the monopoly
- Monopolist is happy to serve everyone as long as $WTP \geq MC$
- Output expands to Q_{PC} (efficient level!)
- No deadweight loss—market is efficient
- But all surplus goes to the producer

Key Point

Under perfect price discrimination:

- Entire surplus is transferred to the monopoly
- Market is still efficient (welfare maximized)
- $\pi_{\max} =$ Total Surplus
- No DWL, but extreme redistribution

Why is perfect price discrimination impossible to implement?

1. **Information problem**: Firm does not know each consumer's WTP
2. **Resale**: Low-WTP consumers could buy and resell to high-WTP consumers

15.6.2 Imperfect Price Discrimination

Since perfect price discrimination is impossible, firms implement less sophisticated types of price discrimination.

15.6.3 Market Separation (Third-Degree Price Discrimination)

Firms separate consumers into groups with different demand curves:

- Senior vs. non-senior
- Student vs. adult
- Business vs. vacation travelers

Each group has its own demand curve. The firm chooses P_1, P_2 to maximize total profit:

- Set $MR_1 = MC$ for market 1 $\Rightarrow$ get Q_1, P_1
- Set $MR_2 = MC$ for market 2 $\Rightarrow$ get Q_2, P_2
- Charge higher price to the group with less elastic demand

15.6.4 Non-Linear Pricing (Second-Degree Price Discrimination)

Non-linear pricing exploits diminishing marginal value:

- 8 oz coffee: \$1.60
- 16 oz coffee: \$2.00
- WTP for additional 8 oz is not as high as for the first 8 oz
- Decreasing willingness to pay by amount consumed

Two common examples:

1. **Two-part pricing**: Fixed fee + per-unit charge
2. **Quantity discounts**: Lower price per unit for larger quantities

Example 15.2 (Two-Part Pricing)**.** Suppose demand is $P = 3 - 0.1Q$ and $MC = AC = 1$.
With linear pricing (standard monopoly):

- $MR = 3 - 0.2Q$
- Set $MR = MC$: $3 - 0.2Q = 1 \Rightarrow Q_M = 10$
- $P_M = 3 - 0.1(10) = \$2$
- $\pi = (2 - 1) \times 10 = \$10$

With two-part pricing:

- Charge $P = MC = \$1$ per unit
- Consumer surplus at $P = 1$: $CS = \frac{1}{2}(3 - 1)(20) = \$40$
- Charge \$40 as a fixed fee (membership/entry fee)
- Consumer buys 20 units at \$1 each
- Total profit: \$40 (from fee) + \$0 (from sales at MC) = \$40

Two-part pricing captures all consumer surplus as profit!

Note

Two-part pricing creates an implicit quantity discount:

- Buy 10 units: \$40 fee + \$10 = \$50 total ⇒ \$5/unit
- Buy 20 units: \$40 fee + \$20 = \$60 total ⇒ \$3/unit

This reduces the effective marginal price for larger quantities.

Other strategies of price discrimination:

- Multi-product pricing

- Requiring a specific complement / bundling

Example 15.3 (Market Separation: Worked Problem)**.** Given:

$$TC = 5Q + 100, \quad MC = 5, \quad AC = 5 + \frac{100}{Q}$$
$$Q_1 = 55 - P_1 \quad \Rightarrow \quad P_1 = 55 - Q_1$$
$$Q_2 = 70 - 2P_2 \quad \Rightarrow \quad P_2 = 35 - 0.5Q_2$$

With market separation (price discrimination):
For Market 1:

$$TR_1 = P_1 \times Q_1 = (55 - Q_1)Q_1 = 55Q_1 - Q_1^2$$
$$MR_1 = 55 - 2Q_1$$

Set $MR_1 = MC$: $55 - 2Q_1 = 5 \Rightarrow Q_1^* = 25, P_1^* = 30$
For Market 2:

$$TR_2 = P_2 \times Q_2 = (35 - 0.5Q_2)Q_2 = 35Q_2 - 0.5Q_2^2$$
$$MR_2 = 35 - Q_2$$

Set $MR_2 = MC$: $35 - Q_2 = 5 \Rightarrow Q_2^* = 30, P_2^* = 20$
Total: $Q = 25 + 30 = 55$

$$TR = 25 \times 30 + 30 \times 20 = 750 + 600 = 1350$$
$$TC = 5(55) + 100 = 375$$
$$\pi = 1350 - 375 = \$975$$

Without price discrimination (single market):
Combined demand: $Q = Q_1 + Q_2 = 55 - P + 70 - 2P = 125 - 3P$
So $P = \frac{125-Q}{3}$

$$TR = P \times Q = \frac{125Q - Q^2}{3}$$
$$MR = \frac{125 - 2Q}{3}$$

Set $MR = MC$: $\frac{125-2Q}{3} = 5 \Rightarrow 125 - 2Q = 15 \Rightarrow Q^* = 55$
$P^* = \frac{125-55}{3} = \$23.33$
$TR = 55 \times 23.33 = 1283.3$, $TC = 375$, $\pi = \$908.33$
Conclusion: Monopoly profit with market separation (\$975) > Monopoly profit in single market (\$908.33)

15.7 Natural Monopolies and Regulation

Definition 15.4 (Natural Monopoly)**.** A **natural monopoly** exists when average cost falls over the entire range of output. This typically occurs with:

- Huge infrastructure investments and fixed costs
- Utility companies (electricity, water, gas)
- Network industries

15.7.1 How to Regulate a Monopoly?

Marginal Cost Pricing Set $P = MC$ to achieve allocative efficiency.
Problem: For a natural monopoly, $MC < AC$ (since AC is falling). Setting $P = MC$ means $P < AC$, so the firm earns economic losses and will exit.

Average Cost Pricing Set $P = AC$ so the firm breaks even ($\pi = 0$).
Problem: Hard to determine true AC (firm might inflate costs). Also results in some allocative inefficiency since $P > MC$.

Two-Tier Pricing

- For $Q \in (0, Q_M]$: Charge monopoly price
- For $Q \in (Q_M, Q_{PC}]$: Charge MC pricing

This allows the firm to earn some profit while expanding output toward the efficient level.

Rate of Return Regulation Allow the firm a "fair" rate of return on capital investment.
Problem: What is a fair rate? If too high, firms over-invest in capital.

16 Imperfect Competition

16.1 Market Structure Spectrum

	Perfect Competition	Imperfect Competition	Monopoly
Firms	Many	Few to many	Single
Pricing	Price taker	Price maker	Price maker
Profit	$\pi = 0$	$\pi \geq 0$ (usually)	$\pi > 0$
Entry	Free entry/exit	Significant barriers	No entry
Product	Homogeneous	Differentiated	Single good

Imperfect competition includes:

- **Monopolistic competition**: Many firms, differentiated products, free entry
- **Oligopoly**: Few firms, strategic interaction (very important!)

16.2 Oligopoly

Definition 16.1 (Oligopoly)**.** An **oligopoly** is a market with 2 or more firms where strategic interaction matters. A **duopoly** is an oligopoly with exactly 2 firms.

If firms cooperate and act like a single monopoly, this is called a **cartel**.

There is no single oligopoly model—many models exist. We will consider three classic oligopoly models that involve strategic interaction:

1. **Cournot model**: Firms simultaneously choose quantities
2. **Bertrand model**: Firms simultaneously choose prices
3. **Stackelberg model**: Sequential quantity choice (leader-follower)

The outcomes differ:

- Cournot: Output and price between monopoly and perfect competition
- Bertrand: Competitive outcome ($P = MC$)
- Stackelberg: Leader advantage, output between Cournot and competitive

16.3 Cournot Model

16.3.1 Setup

- Two firms A and B (identical)
- Firms **simultaneously choose quantities** q_A and q_B
- Produce the same (homogeneous) product
- Market demand: $Q = 120 - P$, equivalently $P = 120 - Q = 120 - q_A - q_B$
- $MC = 0$ for simplicity

Each firm's output depends on what it expects the other firm to produce—similar to Tragedy of the Commons.

16.3.2 Firm A's Problem

Firm A's total revenue:

$$\begin{aligned} TR_A &= P \times q_A = (120 - q_A - q_B) \times q_A \\ &= 120q_A - q_A^2 - q_A q_B \end{aligned}$$

Marginal revenue for A:

$$MR_A = \frac{\partial TR_A}{\partial q_A} = 120 - 2q_A - q_B$$

Set $MR_A = MC_A = 0$:

$$120 - 2q_A - q_B = 0$$

$$q_A^* = 60 - \frac{q_B}{2}$$

This is **Firm A's best response function**—it tells A's optimal output for any given q_B.

16.3.3 Firm B's Problem

By symmetry:

$$q_B^* = 60 - \frac{q_A}{2}$$

This is **Firm B's best response function**.

16.3.4 Nash Equilibrium

At Nash equilibrium, both firms are playing best responses:

$$q_A^* = 60 - \frac{q_B^*}{2}$$
$$q_B^* = 60 - \frac{q_A^*}{2}$$

Substitute:

$$q_A^* = 60 - \frac{1}{2}\left(60 - \frac{q_A^*}{2}\right) = 60 - 30 + \frac{q_A^*}{4}$$
$$\frac{3q_A^*}{4} = 30 \Rightarrow q_A^* = 40$$

By symmetry: $q_B^* = 40$

16.3.5 Cournot Equilibrium Outcome

$$Q = q_A^* + q_B^* = 40 + 40 = 80$$
$$P = 120 - 80 = \$40$$
$$\pi_A = 40 \times 40 - 0 = \$1600$$
$$\pi_B = 40 \times 40 - 0 = \$1600$$
$$\pi_A + \pi_B = \$3200$$

16.3.6 Comparison with Other Market Structures

Perfect competition: $P = MC = 0$

$$P = 0 \Rightarrow Q = 120, \quad \pi = 0$$

Monopoly: $MR = MC$

$$TR = (120 - Q)Q = 120Q - Q^2$$
$$MR = 120 - 2Q = 0 \Rightarrow Q_M = 60$$
$$P_M = 60, \quad \pi_M = 60 \times 60 = \$3600$$

Key Point

Comparison of market structures (with $P = 120 - Q$, $MC = 0$):

	Output	Price	Total Profit
Perfect Competition	120	\$0	\$0
Cournot Duopoly	80	\$40	\$3,200
Monopoly (Cartel)	60	\$60	\$3,600

Cournot outcome lies between perfect competition and monopoly.

16.3.7 Can Firms Collude and Act Like a Monopoly?

Answer: No—collusion is not stable.

Consider what happens when firms try to collude:

q_A	q_B	Q	P	π_A	π_B
30	30	60	60	1800	1800
35	30	65	55	1925	1650
35	35	70	50	1750	1750
40	35	75	45	1800	1575
40	40	80	40	1600	1600
45	40	85	35	1575	1400

The highlighted row shows the Nash equilibrium. Notice that if both firms agree to produce 30 each (monopoly outcome), each earns \$1800. But firm A has an incentive to cheat by producing 35, earning \$1925 while firm B only gets \$1650.

Iterative process to Nash equilibrium:

Given best response functions $q_A^* = 60 - \frac{q_B}{2}$ and $q_B^* = 60 - \frac{q_A}{2}$:

- If $q_B = 30 \Rightarrow q_A^* = 45$
- If $q_A = 45 \Rightarrow q_B^* = 37.5$
- If $q_B = 37.5 \Rightarrow q_A^* = 41.25$
- If $q_A = 41.5 \Rightarrow q_B^* = 39.375$
- If $q_B = 39.375 \Rightarrow q_A^* = 40.3125$
- $\vdots$
- Converges to $q_A^* = 40$, $q_B^* = 40$

16.4 Bertrand Model

In the **Bertrand model**, firms compete on **price** rather than quantity.

16.4.1 Assumptions

- Two firms (A and B) produce **identical products**
- Marginal costs are **constant**
- "Winner takes all": the lower-priced firm gets the entire market demand
- If $P_A = P_B$, firms share the market equally

16.4.2 Equilibrium

Firms try to keep their price lower than the other firm. This undercutting continues until:

$$P = MC \quad \text{and} \quad \pi = 0$$

This leads to the **exact same competitive equilibrium** as perfect competition!

Key Point

Cournot vs. Bertrand Competition:

	Cournot	**Bertrand**
Strategic Variable	Quantity	Price
Equilibrium Price	$P > MC$	$P = MC$
Equilibrium Profit	$\pi > 0$	$\pi = 0$

In both cases, firms would be better off by cooperating, but cooperation is **not stable** since both firms have an incentive to cheat.

Note

Oligopoly outcomes span a huge range—from perfect competition (Bertrand) to monopoly (successful collusion). The actual outcome depends on the nature of competition.

16.5 Bertrand with Product Differentiation

When firms produce **differentiated products**, the Bertrand model yields different results.

16.5.1 Demand Functions

Each firm's demand depends on both prices:

$$q_A = \frac{1}{2} - P_A + P_B$$
$$q_B = \frac{1}{2} - P_B + P_A$$

Note

Each firm's demand is:

- **Decreasing** in its own price
- **Increasing** in the rival's price

16.5.2 Best Response Functions

Firms derive best response functions by maximizing profit with respect to their own price.

The best response functions are upward sloping: when the rival raises its price, the firm optimally responds by also raising its price.

16.5.3 Nash Equilibrium

At the Nash equilibrium:

$$P_A^* = P_B^* > P_{PC}$$

where P_{PC} is the perfectly competitive price (equal to marginal cost).

Key Point

With product differentiation, there is a **trade-off** between quantity (q_A) and price (P_A). Firms can charge prices above marginal cost and earn positive profits, unlike in the homogeneous Bertrand model.

16.6 Hotelling Model: Location as Product Differentiation

The **Hotelling model** (also called the "beach model") illustrates how **location** serves as a form of product differentiation.

16.6.1 Setup

- Consumers are uniformly distributed along a line (e.g., a beach)
- Two sellers (A and B) choose locations on this line
- Consumers incur transportation costs to reach a seller

16.6.2 Two Opposing Effects

1. Direct Effect:

- Firms want to locate near the greatest number of consumers
- This pushes firms toward producing **similar products** (locating near each other)

2. Strategic Effect:

- Locating near each other **toughens price competition**
- Price gets closer to marginal cost
- But locating **further apart** (product differentiation) allows firms to **increase prices**

16.6.3 Other Considerations

Search Costs:

- If consumers are not fully informed about prices, prices can differ even for identical goods
- If you need to pay time or money to search for information on prices, firms can exploit this

Price Discrimination:

- Firms can price discriminate based on **low-cost vs. high-cost consumers**
- The opportunity cost of time differs across consumers
- Examples: coupons, promotional codes (target price-sensitive consumers who have time to search)

16.7 Tacit Collusion

Can firms achieve collusion through **tacit understanding** instead of explicit agreements?

16.7.1 Bertrand Model Outcomes

	Price	**Profit**
Nash Equilibrium	$P = MC$	$\pi = 0$
Collusion	$P = P_M$	Share $\pi_M > 0$

16.7.2 Repeated Game Analysis

Finite time horizon: With a known end date, $P = MC$ (collusion breaks down by backward induction). Promises to maintain collusion are **not credible**.

Infinite/Unknown time horizon: Update prices every period for an unknown number of times.

16.7.3 Payoffs

Let π_M be the monopoly profit and γ be the probability the game continues (or discount factor).

If you collude: Earn $\frac{\pi_M}{2}$ each period (split monopoly profit)

If you cheat: Earn π_M for one period, then 0 in all subsequent periods (trigger strategy punishment)

16.7.4 Condition for Collusion

Collusion gain (present value):

$$\frac{\pi_M}{2}(1 + \gamma + \gamma^2 + \cdots) = \frac{\pi_M}{2} \cdot \frac{1}{1-\gamma}$$

Cheating gain:

$$\pi_M + 0 + 0 + \cdots = \pi_M$$

Collusion is sustainable if:

$$\frac{\pi_M}{2} \cdot \frac{1}{1-\gamma} > \pi_M$$

Solving:

$$\begin{aligned} \frac{1}{2(1-\gamma)} &> 1 \\ 1 &> 2(1-\gamma) \\ 1 &> 2 - 2\gamma \\ 2\gamma &> 1 \\ \gamma &> \frac{1}{2} \end{aligned}$$

Key Point

Collusion is sustainable when $\gamma > \frac{1}{2}$.
The **more patient** the firms are (higher γ), the more likely they are to maintain the cooperative agreement.

16.7.5 Factors Affecting Collusion Sustainability

Interest rates: If the interest rate is high, firms are **less patient** and more likely to break the agreement.

Number of firms: With N firms, collusion is sustainable when:

$$\gamma > 1 - \frac{1}{N}$$

Number of Firms (N)	Minimum γ for Collusion
2	$\gamma > \frac{1}{2}$
3	$\gamma > \frac{2}{3}$
10	$\gamma > \frac{9}{10}$

Key Point

As N increases, it becomes **increasingly more difficult** to sustain collusion. More firms means a more likely competitive outcome.

16.8 Oligopoly Entry and Exit Decisions

16.8.1 Entry Decisions: Perfect Competition vs. Oligopoly

Perfect competition: Entry/exit decisions depend entirely on market price and firm's cost.

Oligopoly: Entry decisions are more complex:

- If I decide to enter, how will that affect the market price in future periods?
- How will competitors respond to my entry?

16.8.2 Commitment and Sunk Costs

A firm's **commitment to entry** often involves **sunk costs**—capital investments that cannot be reversed.

16.9 Stackelberg Model

The **Stackelberg model** is a sequential-move version of Cournot competition.

16.9.1 Setup

- Two firms: A and B
- $MC = AC = 0$
- Demand: $P = 120 - q_A - q_B$
- **Firm A moves first** and commits to production q_A
- **Firm B observes** A's output and then decides q_B

16.9.2 Cournot Review (Simultaneous)

In the simultaneous-move Cournot model:

$$TR_A = q_A \times P = q_A(120 - q_A - q_B) = 120q_A - q_A^2 - q_A q_B$$
$$TR_B = 120q_B - q_B^2 - q_A q_B$$

Setting $MR = MC = 0$:

$$MR_A = 120 - 2q_A - q_B = 0 \Rightarrow q_A^* = \frac{120 - q_B}{2}$$
$$MR_B = 120 - 2q_B - q_A = 0 \Rightarrow q_B^* = \frac{120 - q_A}{2}$$

Cournot equilibrium: $q_A^* = q_B^* = 40$, with $\pi_A = \pi_B = \$1600$.

16.9.3 Stackelberg Solution (Sequential)

Step 1: Solve for B's best response

Firm B observes q_A and maximizes profit:

$$TR_B = (120 - q_A - q_B) \times q_B = 120q_B - q_A q_B - q_B^2$$
$$MR_B = 120 - q_A - 2q_B = 0$$

Firm B's best response:

$$q_B^* = \frac{120 - q_A}{2}$$

Step 2: Firm A incorporates B's response

Firm A **knows** B's best response, so A can substitute it into A's own problem:

$$P = 120 - q_A - q_B = 120 - q_A - \frac{120 - q_A}{2}$$

Simplifying:

$$P = 120 - q_A - 60 + \frac{q_A}{2} = 60 - \frac{q_A}{2}$$

Firm A's total revenue:

$$TR_A = P \times q_A = \left(60 - \frac{q_A}{2}\right) q_A = 60q_A - \frac{q_A^2}{2}$$

Firm A's marginal revenue:

$$MR_A = \frac{dTR_A}{dq_A} = 60 - q_A$$

Setting $MR_A = MC = 0$:

$$60 - q_A = 0 \Rightarrow q_A^* = 60$$

Step 3: Find B's output

$$q_B^* = \frac{120 - 60}{2} = 30$$

16.9.4 Stackelberg Equilibrium Outcome

$$q_A^* = 60, \quad q_B^* = 30$$
$$Q = q_A + q_B = 90$$
$$P = 120 - 90 = 30$$
$$\pi_A = 30 \times 60 - 0 = \$1800$$
$$\pi_B = 30 \times 30 - 0 = \$900$$

16.9.5 Comparison: Cournot vs. Stackelberg

Key Point

	q_A	q_B	Q	P	π_A, π_B
Cournot	40	40	80	\$40	\$1600, \$1600
Stackelberg	60	30	90	\$30	\$1800, \$900

The **first-mover advantage**: By committing to a higher output first, Firm A earns higher profit (\$1800 vs. \$1600) while Firm B earns less (\$900 vs. \$1600).

16.10 Entry Deterrence

Key idea: If Firm A can determine B's output by moving first, could Firm A commit to an output level that would **deter B from entering**?

16.10.1 Setup with Fixed Costs

Now suppose both firms face a **fixed cost** of $F = 785$.

If B does not produce ($q_B = 0$), then:

$$q_A = 120 \Rightarrow P = 0 \Rightarrow \pi_A = 0$$

Outcome when both enter (Stackelberg):

$$q_A = 60, \quad q_B = 30, \quad P = 30$$
$$\pi_A = TR_A - TC = 1800 - 785 = \$1015$$
$$\pi_B = TR_B - TC = 900 - 785 = \$115$$

16.10.2 Entry-Deterring Strategy

To deter entry, Firm A needs to make $\pi_B \leq 0$.

Condition for entry deterrence:

$$\pi_B = P \times q_B - 785 < 0$$

$$P \cdot q_B < 785$$

Substituting $P = 120 - q_A - q_B$ and $q_B = \frac{120 - q_A}{2}$:

$$(120 - q_A - q_B) \cdot q_B < 785$$

$$\left(120 - q_A - \frac{120 - q_A}{2}\right) \cdot \frac{120 - q_A}{2} < 785$$

$$\left(60 - \frac{q_A}{2}\right) \cdot \frac{120 - q_A}{2} < 785$$

$$(60 - 0.5q_A)^2 < 785$$

$$60 - 0.5q_A < \sqrt{785} \approx 28.01$$

$$31.99 < 0.5q_A$$

$$q_A > 63.96$$

16.10.3 Entry Deterrence Outcome

If Firm A produces $q_A = 64$:

$$q_B = \frac{120 - 64}{2} = 28$$
$$P = 120 - 64 - 28 = 28$$
$$\pi_B = 28 \times 28 - 785 = 784 - 785 = -1 < 0$$

Since $\pi_B < 0$, **Firm B does not enter.**
Firm A's profit with entry deterrence:

$$\pi_A = 64 \times 56 - 785 = 3584 - 785 = \$2799$$

(Note: If B doesn't enter, $P = 120 - 64 = 56$)

Key Point

Entry deterrence comparison:

Strategy	π_A	π_B
Allow entry (Stackelberg)	\$1015	\$115
Deter entry ($q_A = 64$)	\$2799	-1 (no entry)

By committing to a slightly higher output ($q_A = 64$ instead of 60), Firm A can deter entry entirely and earn **much higher profit** (\$2799 vs. \$1015).