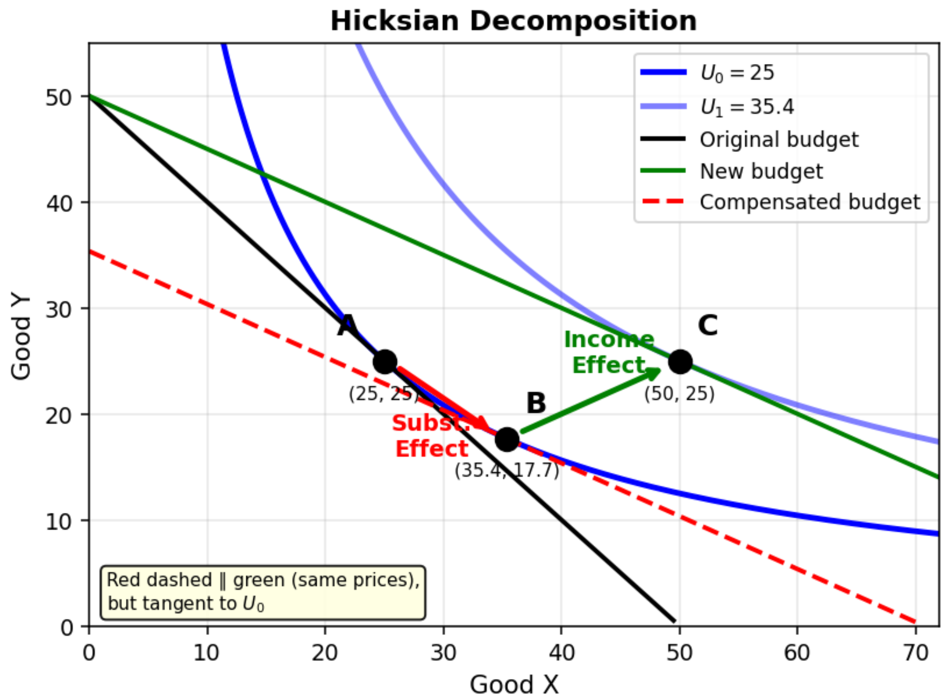
Substitution and Income Effects: A Visual Guide

The Intuition: The Mischievous Economist

When the price of X falls, two things happen simultaneously: (1) X becomes *relatively* cheaper compared to Y, and (2) your purchasing power increases. To separate these effects, imagine a mischievous economist who pickpockets exactly enough money to keep you at your original utility level. You substitute toward X purely because it's relatively cheaper—not because you're richer. That movement is the **substitution effect**. When the money is returned, you get richer and adjust further—that's the **income effect**.



Setup for Numerical Example

Utility function: $U(x, y) = x^{0.5} y^{0.5}$ | **Income:** I = \$100 | **Price of Y:** P_Y = \$2 (constant)
Price of X: Falls from P_X = \$2 to P_X = \$1

Point	Bundle (x, y)	Utility	Description
A (Initial)	(25, 25)	U■ = 25	Original optimum at old prices
B (Compensated)	(35.4, 17.7)	U■ = 25	Same utility, new prices
C (Final)	(50, 25)	U■ = 35.4	New optimum at new prices

Decomposing the Total Effect

Total effect on X: 50 – 25 = **+25 units**
Substitution effect (A→B): 35.4 – 25 = **+10.4 units** (always toward the cheaper good)
Income effect (B→C): 50 – 35.4 = **+14.6 units** (X is a normal good)

Key Takeaways

- The **substitution effect** is *always* toward the cheaper good—this is the source of the Law of Demand for compensated demand.
- The **income effect** depends on whether the good is normal (+) or inferior (–).
- For **normal goods**: both effects reinforce each other → demand curves slope down.
- The **compensated budget** (red dashed) has the same slope as the new budget but stays tangent to U_0.

Mathematical Derivation of Optimal Bundles

Step 1: Deriving the MRS

For the Cobb-Douglas utility function $U(x, y) = x^{\alpha} y^{\beta}$, the marginal utilities are:

$MU_x = \partial U/\partial x = \alpha \cdot x^{\alpha-1} \cdot y^{\beta}$ $\quad$ $MU_y = \partial U/\partial y = \beta \cdot x^{\alpha} \cdot y^{\beta-1}$

The Marginal Rate of Substitution (MRS) is:

$MRS = MU_x / MU_y = (\alpha \cdot x^{\alpha-1} \cdot y^{\beta}) / (\beta \cdot x^{\alpha} \cdot y^{\beta-1}) = \mathbf{(\alpha/\beta) \cdot (y/x)}$

For our example with $\alpha = \beta = 0.5$: **MRS = y/x**

Step 2: The Optimality (Tangency) Condition

At the consumer's optimum, the indifference curve is tangent to the budget line:

$\mathbf{MRS = P_x / P_y}$ ■ $\mathbf{y/x = P_x / P_y}$

Step 3: Solving for Each Bundle

Point A (Original Optimum): $P_x = 2$, $P_y = 2$, $I = 100$

Tangency: $y/x = 2/2 = 1$ ■ $y = x$

Budget: $2x + 2y = 100$ ■ $2x + 2x = 100$ ■ $x = 25$

Solution: $\mathbf{(x_A, y_A) = (25, 25)}$ with $U_0 = \sqrt{25} \cdot \sqrt{25} = \mathbf{25}$

Point B (Compensated Bundle): New prices $P_x = 1$, $P_y = 2$, but must stay on $U_0 = 25$

Tangency with new prices: $y/x = 1/2$ ■ $y = x/2$

Stay on original indifference curve: $\sqrt{x} \cdot \sqrt{y} = 25$

Substitute: $\sqrt{x} \cdot \sqrt{(x/2)} = 25$ ■ $x/\sqrt{2} = 25$ ■ $x = 25\sqrt{2} \approx 35.36$

Solution: $\mathbf{(x_B, y_B) = (35.36, 17.68)}$ with $U = \mathbf{25}$ (unchanged)

Compensated income: $I' = 1(35.36) + 2(17.68) =$ **\$70.71** (the "pickpocketed" amount is \$29.29)

Point C (Final Optimum): $P_x = 1$, $P_y = 2$, $I = 100$ (full income restored)

Tangency: $y/x = 1/2$ ■ $y = x/2$

Budget: $1 \cdot x + 2 \cdot y = 100$ ■ $x + x = 100$ ■ $x = 50$

Solution: $\mathbf{(x_C, y_C) = (50, 25)}$ with $U_1 = \sqrt{50} \cdot \sqrt{25} = \mathbf{35.36}$

Summary of Effects

Effect	Movement	Change in X	Interpretation
Substitution	A → B	+10.4	Pure price effect (utility held constant)
Income	B → C	+14.6	Pure income effect (prices held constant)
Total	A → C	+25.0	Observed change in demand

Why does this matter? The decomposition shows that demand curves slope downward for two distinct reasons: (1) the substitution effect always pushes toward the cheaper good, and (2) for normal goods, the income effect reinforces this. Only for strongly inferior goods (Giffen goods) can the income effect overwhelm the substitution effect and produce an upward-sloping demand curve.