# Problem 1: Taxation and Consumer Welfare

## EC 224 – Intermediate Microeconomics

**Setup:** Kate consumes chips ($C$) and soda ($S$) with utility function $U = \sqrt{C \times S}$. Both goods cost \$1 per unit ($P_C = P_S = 1$) and her income is $M = 150$. The marginal rate of substitution is $MRS = S/C$.

(i) **Find equilibrium demand and utility (no tax).**

At the consumer's optimum, $MRS = P_C/P_S$:

$$\frac{S}{C} = \frac{1}{1} \implies S = C$$

Substitute into the budget constraint $C + S = 150$:

$$2C = 150 \implies C^* = 75, \quad S^* = 75$$

Utility:

$$U = \sqrt{75 \times 75} = 75$$

(ii) **A \$0.60 per-unit tax is imposed on soda. Find new demands, utility, and government revenue.**

The new price of soda is $P_S = 1 + 0.60 = 1.60$.

Optimality condition:
$$\frac{S}{C} = \frac{P_C}{P_S} = \frac{1}{1.6} \implies C = 1.6\,S$$

Substitute into the new budget constraint $C + 1.6\,S = 150$:

$$1.6\,S + 1.6\,S = 150 \implies 3.2\,S = 150 \implies S^* = 46.875$$

$$C^* = 1.6 \times 46.875 = 75$$

Utility:
$$U = \sqrt{75 \times 46.875} = \sqrt{3515.625} \approx 59.29$$

Government revenue:
$$R = 0.60 \times 46.875 = \$28.125$$

(iii) **An income tax raises the same revenue (\$28.125). Find new demands and utility.**

New income after the lump-sum tax: $M = 150 - 28.125 = 121.875$.

Prices are unchanged ($P_C = P_S = 1$), so the optimality condition is the same as part (i):

$$S = C$$

Budget constraint:
$$C + S = 121.875 \implies C^* = S^* = 60.9375$$

Utility:
$$U = \sqrt{60.9375 \times 60.9375} = 60.9375$$

**Key Takeaway:** For the same government revenue of \$28.125, Kate's utility is **higher under the income tax** (60.94) than under the per-unit tax on soda (59.29). The excise tax distorts relative prices, creating a deadweight loss that makes the consumer worse off compared to a lump-sum tax that raises the same revenue.